(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

SECOND YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: MAA 212

COURSE TITLE: DYNAMICS I

DATE: 14/6/2021 **TIME:** 2:00 P.M - 4:00 P.M

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 4 Printed Pages. Please Turn Over.

QUESTION ONE COMPULSORY (30 MARKS)

a) Define the terms:
   i) Scalar Product (1 mark)
   ii) Scalar Triple Product (1 mark)

b) State four properties of scalar triple product (4 marks)

c) Prove that the projection of $\vec{A}$ on $\vec{B}$ is equal to $\vec{A} \bullet \underset{\sim}{b}$ where $\underset{\sim}{b}$ is a unit vector in the direction of $\vec{B}$ (4 marks)

d) The vertices of a triangle are located at $A(6,-1,2)$, $B(-2,3,-4)$ and $C(-3,1,5)$. Find the angle at vertex A (5 marks)

e) In each case, determine whether the following vectors are linearly dependent or linearly independent:

   i) $\vec{A} = 2\underset{\sim}{i} + \underset{\sim}{j} - 3\underset{\sim}{k}$, $\vec{B} = \underset{\sim}{i} - 4\underset{\sim}{k}$ and $\vec{C} = 4\underset{\sim}{i} + 3\underset{\sim}{j} - \underset{\sim}{k}$ (4 marks)

   ii) $\vec{A} = \underset{\sim}{i} - 3\underset{\sim}{j} + 2\underset{\sim}{k}$, $\vec{B} = 2\underset{\sim}{i} - 4\underset{\sim}{j} - \underset{\sim}{k}$ and $\vec{C} = 3\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}$ (4 marks)

f) Evaluate the angle between the following two vectors:

   i) $\vec{a} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 5 \\ 2 \\ 3 \end{pmatrix}$ (4 marks)

   ii) $\vec{r} = \begin{pmatrix} 7 \\ 2 \\ -1 \end{pmatrix}$ and $\vec{t} = \begin{pmatrix} 1 \\ 4 \\ 5 \end{pmatrix}$ (3 marks)

QUESTION TWO (20 MARKS)

a) Find the work done in moving an object along the vector $\vec{r} = 3\underset{\sim}{i} + 2\underset{\sim}{j} - 5\underset{\sim}{k}$ if the force applied is $\vec{F} = 2\underset{\sim}{i} - \underset{\sim}{j} - \underset{\sim}{k}$. (3 marks)

b) If $\vec{R} = 2\underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k}$, $\vec{S} = \underset{\sim}{i} + 3\underset{\sim}{j} - 2\underset{\sim}{k}$, $\vec{T} = -2\underset{\sim}{i} + \underset{\sim}{j} - 3\underset{\sim}{k}$ and $\vec{U} = 3\underset{\sim}{i} + 2\underset{\sim}{j} + 5\underset{\sim}{k}$, find the scalars a, b and c such that $\vec{U} = a\vec{R} + b\vec{S} + c\vec{T}$ (5 marks)

c) Find a unit tangent vector on the curve $x = t^2 + 1$, $y = 4t - 3$, $z = 2t^2 - 6t$ and determine the unit tangent at a point where $t = 2$ (3 marks)

d) i) Find the value a so that $\vec{A} = 2\underset{\sim}{i} + a\underset{\sim}{j} + \underset{\sim}{k}$ and $\vec{B} = 4\underset{\sim}{i} - 2\underset{\sim}{j} - 2\underset{\sim}{k}$ are

perpendicular. (2 marks)

ii) Show that $\left|\vec{A}\times\vec{B}\right|^2+\left|\vec{A}\bullet\vec{B}\right|^2=\left|\vec{A}\right|^2\left|\vec{B}\right|^2$ (2 marks)

e) The position vector of a particle P at time t is given by:

$\vec{r}=(2t^2-3)\underset{\sim}{i}+(4t+4)\underset{\sim}{j}+(t^3+2t^2)\underset{\sim}{k}$. Find:

i) the distance OP when $t=0$ (3 marks)
ii) the velocity of P when $t=1$ (1 mark)
iii) the acceleration of P when $t=2$ (1 mark)

QUESTION THREE (20 MARKS)

a) Find the projection of vector $\vec{A}=\underset{\sim}{i}-2\underset{\sim}{j}+\underset{\sim}{k}$ on the vector $\vec{B}=4\underset{\sim}{i}-4\underset{\sim}{j}+7\underset{\sim}{k}$ (3 marks)

b) A particle moves so that its position vector is given by $\vec{r}=\cos\omega t\,\underset{\sim}{i}+\sin\omega t\,\underset{\sim}{j}$ where ω is a constant. Show that the:

i) Velocity $\vec{V}$ of the particle is perpendicular to $\vec{r}$. (2 marks)

ii) Acceleration $\vec{a}$ is directed towards the origin and has a magnitude proportional to the distance from the origin. (3 marks)

iii) $\vec{r}\times\vec{V}=$ a constant (2 marks)

c) A fish swimming in a horizontal plane has a velocity of $V_0=\vec{4}\,\underset{\sim}{i}+\underset{\sim}{j}$ at a point in the ocean where the position vector is $\vec{r}_0=10\,\underset{\sim}{i}-4\,\underset{\sim}{j}$ relative to a stationary rock at the shore. After the fish swims with a constant acceleration in 20 seconds, its velocity $\vec{V}=20\,\underset{\sim}{i}-5\,\underset{\sim}{j}$.

i) What are the components of the acceleration (2 marks)
ii) What is the direction of acceleration with respect to the fixed $x-$axis (2 marks)
iii) Where is the fish at $t=25$seconds? (2 marks)
iv) What is its speed and in what direction is it moving? (4 marks)

QUESTION FOUR (20 MARKS)

a) Find the work done in moving an object along a straight line from $(3,2,-1)$ to $(2,-1,4)$ in a force field given by $\vec{F}=4\,\underset{\sim}{i}-3\,\underset{\sim}{j}+2\,\underset{\sim}{k}$

(4 marks)

b) Evaluate: $\left(2\underset{\sim}{i}-3\underset{\sim}{j}\right)\left[\left(\underset{\sim}{i}+\underset{\sim}{j}-\underset{\sim}{k}\right)\times\left(2\underset{\sim}{i}-\underset{\sim}{k}\right)\right]$ (3 marks)

c) If $\vec{A}=A_1\underset{\sim}{i}+A_2\underset{\sim}{j}+A_3\underset{\sim}{k}$, $\vec{B}=B_1\underset{\sim}{i}+B_2\underset{\sim}{j}+B_3\underset{\sim}{k}$ and $\vec{C}=C_1\underset{\sim}{i}+C_2\underset{\sim}{j}+C_3\underset{\sim}{k}$, show that

$$\vec{A}\bullet\left(\vec{B}\times\vec{C}\right)=\begin{vmatrix}A_1 & A_2 & A_3\\ B_1 & B_2 & B_3\\ C_1 & C_2 & C_3\end{vmatrix}$$ (4 marks)

d) The position in meters of an electron is given by $\vec{r}=3t\underset{\sim}{i}-4t^2\underset{\sim}{j}+2\underset{\sim}{k}$, where t is the time in seconds. Determine:

i) The velocity vector $\vec{V}$ of the electron (2 marks)

ii) The velocity at $t=2$ seconds in unit vector notation (2 marks)

iii) The magnitude and direction of the velocity at $t=2$ seconds (5 marks)

QUESTION FIVE (20 MARKS)

a) A riffle is aimed horizontally at a target 30 m away. The bullet hits the target 1.9 cm below the aiming point.

i) What is the bullet's time of flight? (3 marks)

ii) What is the muzzle's velocity of the riffle? (2 marks)

b) A ball is thrown with a speed of $25m/s$ at an angle of 40^0 above the horizontal directly towards a wall. The wall is $22m$ from the release point of the ball.

i) How long does the ball take to reach the wall? (2 marks)

ii) How far above the release point does the ball hit the wall? (2 marks)

iii)What are the horizontal and vertical components of its velocity as it hits the wall? (3 marks)

c)A rigid body is rotating with angular speed $7rad/s$ about a fixed axis through the points $A(2,3,-1)$ and $B(-4,0,1)$. The rotation is in the left handed screw relative to $\vec{AB}$. Find the angular velocity vector of the body. (3 marks)

d)The motion of a body rotating about an axis is defined by the notation $\theta=3t^3-18t^2+26t+8$, where θ is the angular position expressed in radians and t is time in seconds.

i) Determine when angular velocity is zero (2 marks)

ii) Find the angular position and the total angular distance travelled when the acceleration becomes zero. (3 marks)