KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

FIRST SEMESTER
MAIN EXAMINATIONS

FOR THE DEGREE OF MASTER OF SCIENCE IN PHYSICS

COURSE CODE: SPH 811

COURSE TITLE: MATHEMATICAL PHYSICS

DURATION: 2 HOURS

DATE: 14/06/2021 **TIME: 8-10AM**

INSTRUCTIONS TO CANDIDATES

- Answer **ANY THREE QUESTIONS.**
- Each question carries **20 MARKS.**

- ALL Symbols have their usual meaning
- $\int_0^\infty r^{-1}e^{-r^2}dr = 0$

QUESTION ONE (20 MARKS)

a) Determine the divergence of the vector field $\vec{V}(x_{1,}x_2, x_3) = V_1\hat{e}_1 + V_2\hat{e}_2 + V_3\hat{e}_3$ given that it is differentiable. (3marks)

b) Use the Stokes vector integral theorem to verify the Maxwell's equation of electromagnetism i.e. $\vec{\nabla} \times \vec{B} = \mu_o\vec{J}$ (4marks)

c) A fluid of density $\rho(\vec{r})$ moves with a velocity $\vec{V}(\vec{r})$. Show that if there are no sinks or sources then the following continuity equation is satisfied. $\frac{\partial\rho(\vec{r})}{\partial x} + \vec{\nabla}.(\rho\vec{V}) = 0$. (5marks)

d) Use the calculus of residues to evaluate $\int_0^{2\pi} \frac{\cos 2\theta}{5+4\cos\theta} d\theta$ (5marks)

e) A vector field $\vec{\mathbf{F}} = (2xy + z^3)\mathbf{i} + x^2\mathbf{j} + 3xz^2\mathbf{k}$. Show that the vector field is irrotational. (3marks)

QUESTION TWO (20 MARKS)

a) Determine the eigen values and the corresponding eigen vectors of the matrix (7marks)
$$\begin{pmatrix} 0 & -1 & 1 \\ 2 & -3 & 1 \\ 1 & -1 & -1 \end{pmatrix}$$

b) Show that $B(p,q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$ (7marks)

c) Use the result in (b) above to evaluate $\int_0^5 \omega^5(1-\omega)^4\, d\omega$ (3marks)

d) Show that the vectors $\vec{V}_1 = (2,0,-1)$, $\vec{V}_2 = (0,-1,0)$ and $\vec{V}_3 = (2,0,4)$ in R^3 form an orthogonal set. (3marks)

QUESTION THREE (20 MARKS)

a) Use the gamma function to evaluate $\Gamma\left(\frac{1}{2}\right)$ (9marks)

b) Use the result in (a) above to evaluate $\int_0^\infty x^{\frac{1}{2}}e^{-x^2} dx$ (4marks)

e) Given the vectors $\vec{V}_1 = (2,0,-1)$, $\vec{V}_2 = (0,-1,0)$ and $\vec{V}_3 = (2,0,4)$ in R^3.
   i) Show that they form an orthogonal set (2marks)
   ii) Show that the set is not orthonormal (2marks)
   iii) Form an orthonormal set of vectors (3marks)

QUESTION FOUR (20 MARKS)

a) Use the calculus of residues to show that $\int_0^{2\pi} \frac{d\theta}{a+b\cos\theta} = \frac{2\pi}{\sqrt{a^2-b^2}}$ where $a > b > 0$

(5marks)

b) Obtain the first and second forms of the Greens Theorem. (5marks)

c) Solve the differential equation $y^{//} - 3y^{/} + 2y = e^{3t}$ $given\ y(0) = 1\ and\ y'(0) = 0$ (5marks)

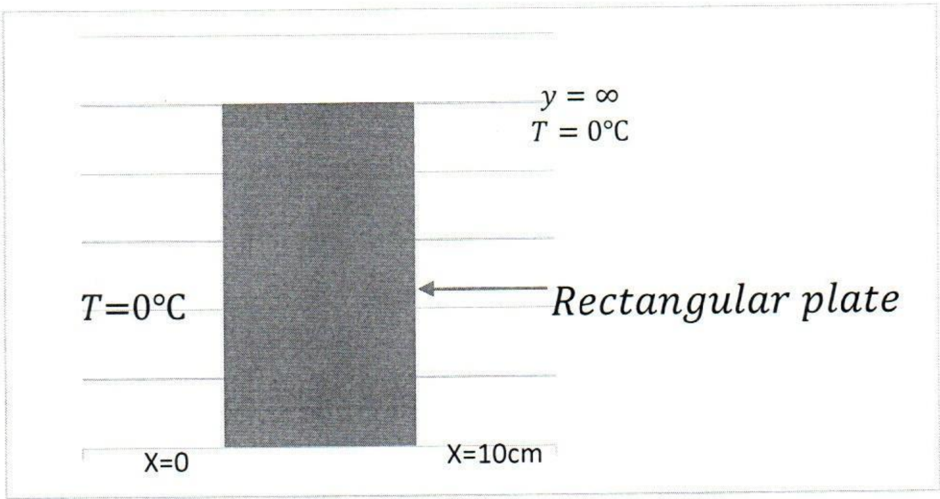
d) Obtain the Fourier series for the periodic function defined as

$$f(x) = \begin{cases} -k & -\pi < x < 0 \\ k & 0 < x < \pi \end{cases}$$ (5marks)

QUESTION FIVE (20 MARKS)

a) Using the Schrodinger equation derive the ground state wave function for a free particle in a one dimensional case. (6marks)

b) A long rectangular plate has its long sides and the far end at $0^{\circ}C$ and the base at $100^{\circ}C$. The width of the plate is 10cm. Find the steady state temperature inside the plate. (8marks)



e) Given that $\vec{V_1} = (2,-1,0)$, $\vec{V_2} = (1,0,-1)$ and $\vec{V_3} = (3,7,-1)$ is a basis of R^3. Find the orthogonal basis by Gram-Schmidt procedure hence determine the orthonormal basis (6marks)

Table 15.2 Laplace Transforms

$f(s)$	$F(t)$	Limitation
1. 1	$\delta(t)$	Singularity at $+0$
2. $\frac{1}{s}$	1	$s > 0$
3. $\frac{n!}{s^{n+1}}$	t^n	$s > 0$ $n > -1$
4. $\frac{1}{s-k}$	e^{kt}	$s > k$
5. $\frac{1}{(s-k)^2}$	te^{kt}	$s > k$
6. $\frac{s}{s^2-k^2}$	$\cosh kt$	$s > k$
7. $\frac{k}{s^2-k^2}$	$\sinh kt$	$s > k$
8. $\frac{s}{s^2+k^2}$	$\cos kt$	$s > 0$
9. $\frac{k}{s^2+k^2}$	$\sin kt$	$s > 0$
10. $\frac{s-a}{(s-a)^2+k^2}$	$e^{at}\cos kt$	$s > a$
11. $\frac{k}{(s-a)^2+k^2}$	$e^{at}\sin kt$	$s > a$
12. $\frac{s^2-k^2}{(s^2+k^2)^2}$	$t\cos kt$	$s > 0$
13. $\frac{2ks}{(s^2+k^2)^2}$	$t\sin kt$	$s > 0$