(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

FIRST YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: **MAA 111**

COURSE TITLE: **DIFFERENTIAL CALCULUS/CALCULUS I**

DATE: 12/5/2021 **TIME:** 2:00 P.M - 4:00P.M

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE COMPULSORY (30 MARKS)

(a) Define the following terms

(i) A function (2 mks)

(ii) Range of a function (2 mks)

(iii) A stationary point (2 mks)

(b) Find the domain and the range of the function $f(x) = \sqrt{x-6}$ (2 mks)

(c) Evaluate the following limits;

(i) $\lim_{x\to 3} \frac{x-3}{\frac{1}{x}-\frac{1}{3}}$ (4mks)

(ii) $\lim_{x\to \infty} \frac{2x^3+4x-3}{\sqrt{x^6-\frac{2}{x}}}$ (3 mks)

(d) Given the function $y = \ln(x^3 - 2x^2 + x)$, determine y^{II} (4 mks)

(e) If $g(x) = \frac{x}{x-5}$ and $h(x) = 2x^2 - x - 3$ find

(i) goh (2 mks)

(ii) hog (2 mks)

(f) (i) State the Rolle's theorem (2 mks)

(ii) Determine whether the function $f(x) = x^2 - 3x + 4$ satisfy the condition ofRolle's theorem on the interval $(0,3)$. If so find the number c that satisfy the conclusion of Rolle's theorem. (5mks)

QUESTION TWO (20 MARKS)

(a) Prove that $\lim_{x\to -3} (\frac{1}{3}x + 3) = 2$ (5mks)

(b) Given $y = 2secx^4$ find $\frac{dy}{dx}$ (4 mks)

(c) Find $\frac{d^2y}{dx^2}$ given that $x(t) = \frac{1}{3}t^3 - \frac{1}{2}t^2 + 2t - 4,\ y(t) = 2t^2 - 4$ (5mks)

(d) Mwangi wishes to enclose his rectangular land with 1200M of barbed wire of which one of the longest side is a wall. Determine the dimension the dimensions that will maximize the area enclosed. (6 mks)

QUESTION THREE (20 MARKS)

(a) Determine whether each of the following functions is even, odd or neither

(i) $f(x) = x^3 + x$ (2 mks)

(ii) $g(x) = 1 - x^6$ (2 mks)

(iii) $h(x) = 2x - 5x^2$ (2 mks)

(b) Given the equation $y = \frac{1}{4}x^4 - \frac{2}{3}x^3 - 4.5x^2 + 18x + 6$

(i) Find all the turning points (5 mks)

(ii) The nature of the turning points (5mks)

(iii) Sketch the graph (4 mks)

QUESTION FOUR (20 MARKS)

(a) If $y = e^{-x}cosx$ prove that $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$ (10 mks)

(b) Prove that $\lim_{x\to 0}\frac{sinx}{x} = 1$ (10 mks)

QUESTION FIVE (20 MARKS

(a) Determine if the following function is continuous at $x = 4$

$$f(x) = \begin{cases} x^2 - 3 & x < 4 \\ 13 & x = 4 \\ \frac{3x+4}{2} + 5 & x > 4 \end{cases}$$ (5 mks)

(b) Find the equation of the normal to the curve $x^2y - 3xy + 7x = 11$ At (1,1) (7 mks)

(c) The position of a stone projected vertically upwards at any time t is given by $S = 2t^2 - \frac{1}{3}t^3 + 10$

(i) Find the maximum height of the stone from the ground (5 mks)

(ii) Acceleration of the stone at $t = 4$ seconds (3 mks)