(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2019/2020 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE (MATHEMATICS)

COURSE CODE: **MAT 402**

COURSE TITLE: TOPOLOGY II

DATE: 11/11/2020 **TIME:** 9 AM -11 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION 1 (30 MARKS)

(a) Define the following

(i) Second countable space **(2mks)**

(ii) T_1 space. **(2mks)**

(iii) Regular space. **(2mks)**

(iv) Connected spaces. **(2mks)**

(v) Sequential compactness **(2mks)**

(b) State without proof the Urysohn's lemma. **(2mks)**

(c) Prove that every subspace of T_1 is T_1. **(10mks)**

(d) Prove that any compact Hausdorf space is normal. **(6mks)**

(e) Define a path in a topological space X. **(2mks)**

QUESTION 2 (20 MARKS)

(a) Prove that T_2 space is a T_1 space but the converse is not true . **(10mks)**

(b) Prove that every sub-space a T_2 space is also a T_2 space. **(10mks)**

QUESTION 3. (20 MARKS)

(a) Prove that an infinite set with co-finite topology is connected. **(6mks)**

(b) Let (X, τ) be a topological space. Show that $= (X, \tau)$ is disconnected if and only if 'X' contains non empty set A which is both open and closed. **(7mks)**

(c) Prove that a space X is connected if and only if there does not exist a surjective continuous function f from X onto the two point discrete space. **(7mks)**

QUESTION 4. (20 MARKS)

(a) Prove that a metric space is compact if and only if it is sequentially compact. **(14mks)**

(b) Let $f: X \to Y$ be a continuous map of topological spaces, with X compact. Prove that f(X) is compact. **(6mks)**

QUESTION 5. (20 MARKS)

Let (X, τ)be a topological space. Show that the following statements are equivalent.

(a) X is T_1-space

(b) Each singleton subset of X is closed.

(c) Each subset A of X is the intersection of its open superset. **(20mks)**