(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

SECOND YEAR FIRST SEMESTER

SPECIAL/SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF MASTER OF SCIENCE IN STATISTICS

COURSE CODE: **STA 844**

COURSE TITLE: **STOCHASTIC PROCESS I**

DATE: 09/10/18 **TIME: 8 AM -11 AM**

INSTRUCTIONS TO CANDIDATES

Answer Question ONE and Any Other TWO Questions

TIME: 3 Hours

Question one (20mks)

a) State what is meant by Markov-chain.

b) Show that $p_s(x_{n+3} = j \mid x_n = i) = {p_{ij}}^3$

c) A six-state Markov-chain with transition matrix

$$T = \begin{bmatrix} 1/2 & 1/2 & 0 & 0 & 0 & 0 \\ 1/4 & 3/4 & 0 & 0 & 0 & 0 \\ 1/4 & 1/4 & 1/4 & 1/4 & 0 & 0 \\ 1/4 & 0 & 1/4 & 1/4 & 0 & 1/4 \\ 0 & 0 & 0 & 0 & 1/2 & 1/2 \\ 0 & 0 & 0 & 0 & 1/2 & 1/2 \end{bmatrix}$$

i. Show that state E_1 is recurrent non-null

ii. Show that state E_3 is transient.

Question two (20mks)

a) Obtain pgf of x if $x \sim N(\mu, \sigma^2$

b) Hence compute :

$$E(x^3 + 2x + 1)$$

Question three (20mks)

a) Define what is meant by probability generating function and show that

$$G^k(1) = E[x(x-1)(x-2)\dots(x-k+1)]$$

b) Given that $x \sim B(n,p)$obtain the pgf hence compute $E(x)$ and $Var(x)$

c) Prove the conditional variance formula

$$Var(x) = E\left[Var\left({}^{x}/_{y}\right)\right] + Var\left[E\left({}^{x}/_{y}\right)\right]$$

bypgf technique.

Question Four (20mks)

a) Define the following terms

   i. A recurrent event

   ii. A transient event

b) Prove the Chapman-Kolmogorov equation

$$P_{ij}{}^{n} = \sum_{k=1} P_{ik}{}^{n-1} P_{kj}$$

c) Let $x = \{x_n; n\ in\ \mathbb{N}\}$ be a Markov chain with state space$E = \{a, b, c\}$ and transition matrix

$$T = \begin{bmatrix} 1/2 & 1/4 & 1/4 \\ 2/3 & 0 & 1/3 \\ 3/5 & 2/5 & 0 \end{bmatrix}$$

Compute: $P_s\{x_1 = b, x_2 = c, x_3 = a, x_5 = a, x_6 = c,\ x_7 = b \,|\, x_0 = c\}$

Question five (20mks)

a) In a branching process, the probability that any individual has j descendants is given by

$$p_0 = 0, \quad p_j = \tfrac{1}{2}j, \quad j = 1$$

Show that the pgf of the first generation is

$$G_n(s) = \tfrac{s}{2-s}$$

Find the generating function $G_2(s),\ G_3(s),\ G_4(s)$. Show by induction that

$$G_n(s) = \tfrac{s}{2^n - (2^n - 1)s}$$

b) Let $G_n(s)$ be the pgf of the population size of the n^{th} generation of branching process. The probability that the population size is zero at the n^{th} generation is$G_n(0)$. What is the probability that the population actually becomes extinct at the n^{th} generation? Where

$$p_j = \frac{1}{2^j + 1} \qquad j = 0,1,2 \ldots$$

$$G_n(s) = \frac{n}{n+1} + \sum_{r=1}^{\infty} \frac{n^{r-1}}{(n+1)^{r+1}} s^r$$

Find the probability, of extinction at the n^{th}generation.