(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

FIRST YEAR SECOND SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF MASTER OF SCIENCE IN STATISTICS

COURSE CODE: **STA 841**

COURSE TITLE: **TIME SERIES I**

DATE: 05/10/18 **TIME:** 8 AM -11 AM

INSTRUCTIONS TO CANDIDATES

Answer Question ONE and Any Other TWO Questions

TIME: 3 Hours

Question 1 Compulsory (30 marks)

a. Explain what you understand by the following terms as used in time series analysis
   i. time series analysis
   ii. Stationary time series in weak sense.
   iii. Filtering
   iv. Analysis of time series in frequency domain
   v. Correlogram (5 marks)

b. Explain the demerits of moving average method in the analysis of time series. (2 marks)

c. Discuss the components of a time series and explain how each one of them is applied in real life situation. (4 marks)

d. The sales of a company in Kshs. For the years 2010 to 2016 are given below:

Year	2010	2011	2012	2013	2014	2015	2016
Sales in Million Kshs.	30	45	63	90	130	188	273

Estimate the sales for the year 2018 using an equation of the form $Y_t = ab^x$, where x = years and y = sales. (6 marks)

e. Consider an AR (2) given by $X_t = \frac{1}{3}X_{t-1} + \frac{2}{9}X_{t-2} + e_t$, where $e_t \sim N(0, \sigma^2)$ is the white noise. Is the process X_t stationary? If so, find its Auto Correlation Function (ACF). (7 marks)

f. Let X_t be a MA (2) given by $X_t = \mu + e_t + \alpha_1 e_{t-1} + \alpha_2 e_{t-2}$, where $e_t \sim N(0, \sigma^2)$ random variables. Show that the ACF does not depend on μ. (6 marks)

Answer any Two questions

Question 2 (20mks)

a. Define a spectral density function. (2mks)

b. Find the spectral density function of the MA (2) model $X_t = 0.5e_{t-2} + 0.8e_{t-1} + e_t$, where $\{e_t\}$ is a white noise process with mean 0 and variance σ^2. hence sketch it.

Describe least squares estimation of parameters of the AR (2) model $X_t = \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + e_t$, $\{e_t\}$ is a white noise process with mean 0 and variance σ^2. (8 mks)

c) Show that the ARMA (1, 1) process $X_t = 0.5X_{t-1} + e_t - 0.3e_{t-1}$ is stationary and invertable. (10 marks)

Question 3 **(20 marks)**

a. Calculate the five year moving average for the following data of the number of commercial industrial failures in a country during 1980 to 1995 and present this information on a graph. Comment on the result. (5 marks)

Year	1980	1981	1982	1983	1984	1985	1986	1987
Number of failures	24	27	29	33	21	13	13	11

Year	1988	1989	1990	1991	1992	1993	1994	1995
Number of failures	10	14	12	15	13	10	04	02

b. Find the ACF of AR (2) given by $X_t = \frac{1}{12}X_{t-1} + \frac{1}{12}X_{t-2} + e_t$, where $e_t \square N\left(0,\sigma^2\right)$.

(7 marks)

c.

i. Define the term differencing as used in time series analysis. (1 mark)

ii. Let $X_t = \beta_0 + \beta_1 X^1 + \beta_2 X^2 + e_t$, with $e_t \square\ iid\ N\left(0,\sigma^2\right)$. Convert X_t into a stationary time series by method of differencing. (7 marks)

Question 4 **(20 marks)**

a. The price of a commodity during 1999 to 2004 is given below:

Year	1999	2000	2001	2002	2003	2004
Price in Kshs.	200	207	228	240	281	292

Fit a parabola $X_t = \beta_0 + \beta_1 X^1 + \beta_2 X^2 + e_t$ to this data. Estimate the price of the commodity for the year 2009. (5 marks)

b. Given an $AR(p)$ as $X_t = \sum_{j=1}^{p} \alpha_j X_{t-j} + e_t$, where α_j are constants, derive the necessary and sufficient condition for the $AR(p)$ to be stationary. (6 marks)

c. Is the time series $Y_t = X_t - X_{t-1}$ stationary, given that $X_t = \beta \sum_{j=1}^{p} e_{t-j} + e_t$. (9 marks)

Question 5 **(20 marks)**

a. Briefly explain the main stages in setting up a Box – Jenkins forecasting models.

(4 marks)

b. Given an AR (1) i.e. $X_t = \alpha X_{t-1} + e_t$, where α is a constant. If $|\alpha| < 1$, show that X_t may be expressed as an infinite order MA process. Find its autocovariance and its autocorrelation functions

c. Given that $Y_t = \sum_{j=-\infty}^{\infty} \alpha_j X_{t-j}$, where $\alpha_j = \begin{cases} \frac{1}{2m+1} & ; j = 0, \pm 1, \pm 2, ..., \pm m \\ 0 & ; \; elsewhere \end{cases}$ is a linear filter, perform filtering on the time series process $X_t = A \sin \lambda t$. (4 marks)

d. Calculate the seasonal indices for the data given by the method of ratio to moving average. (5 marks)

Year	Output of wheat in million tons per quarter			
	I	II	III	IV
1995	75	69	68	70
1996	72	65	63	68
1997	75	70	70	74