(*Knowledge for Development*)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2016/2017 ACADEMIC YEAR

FIRST YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF MASTER OF SCIENCE IN STATISTICS

COURSE CODE: **STA 814**

COURSE TITLE: STATISTICAL INFERENCE I

DATE: 13/9/17 **TIME:** 2 PM -5 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 3 Hours

QUESTION ONE (30 MARKS)

a) Define the following terms

(i) Bias

(ii) Unbiased

(iii) MSE

(iv) Efficiency

(v) Standard error

b) (i) What is an exponential families distribution?

(ii) If $X - b(p,n)$ (binomial) distribution, show that it belongs to the exponential families. Hence obtain the mean and variance of X.

c) Show that the estimator S^2 is a consistent estimator of δ^2

d) Determine the Crammer Rao Lower Bound (CRLB) for the variance of an unbiased estimator of θ in the lognormal distribution

$$f(x;\theta) = \begin{cases} \dfrac{1}{x(2\pi\theta)^{\frac{1}{2}}} \exp\left(-\dfrac{1}{2\theta} In^2 x\right) & for\ x \geq 0\ and\ \theta > 0 \\ 0 & elsewhere \end{cases}$$

QUESTION TWO (20 MARKS)

a) Determine the MLE of θ for

$$f(x;\theta) = \begin{cases} \dfrac{1}{\theta} & for\ 0 \leq x \leq \theta \\ 0 & elsewhere \end{cases}$$

b) Show that the estimator obtained in (a) is biased but consistent

c) If X is a random variable distributed as a gamma distribution with parameters t and λ.

Show that $E(X) = \dfrac{t}{\lambda}$ and $Var(X) = \dfrac{t}{\lambda^2}$. Obtain the estimators of t and λ by the methods of moment estimator (ME).

QUESTION THREE (20 MARKS)

a) Let $\overline{X_1}$ and $\overline{X_2}$ be independent samples means from two normal populations $N(\mu,\delta_1^2)$ and $N(\mu,\delta_2^2)$ respectively. If δ_1^2 and δ_2^2 are known. Show that a 100(1-α)% CI for $\mu_1-\mu_2$ is

$$\Pr\left\{(\overline{X_1}-\overline{X_2})-Z_{\alpha/2}\left(\frac{\delta_1^2}{n_1}+\frac{\delta_2^2}{n_1}\right)^{\frac{1}{2}}<\mu_1-\mu_2<(\overline{X_1}-\overline{X_2})+Z_{\alpha/2}\left(\frac{\delta_1^2}{n_1}+\frac{\delta_2^2}{n_1}\right)^{\frac{1}{2}}\right\}$$

Where n_1 and n_2 are respectively, the sample sizes from $N(\mu,\delta_1^2)$, $N(\mu,\delta_2^2)$ and $Z_{\alpha/2}$ is the value of standardized normal random variable z such that $R\{Z>Z_{\alpha/2}\}=\alpha/2$

b) The fuel consumption of a certain type of vehicle is approximately, normal, with standard deviation 3 miles per litre. If a sample of 64 vehicles has an average fuel consumption of 16 miles per litre

(i) Determine the 95% CI for the mean fuel consumption of all vehicles of this type.

(ii) How large a sample is needed if we wish to be 0.5 miles per litre of the true mean?

QUESTION FOUR (20 MARKS)

a) Let $X\sim B(1,P)$ Obtain the M.L.E of P hence or otherwise the MLE of $p(1-p)$.

b) Let $X_1, X_2, \ldots\ldots\ldots, X_n$ be a random sample from a negative exponential density

$$f(x;\theta)=\begin{cases}\theta e^{-\theta x} & x\geq 0\\ 0 & elsewhere\end{cases}$$

Estimate θ by the ME method

c) Let $X_1, X_2, \ldots\ldots\ldots, X_n$ be a random sample from a normal distribution for which the mean 0 and the variance σ^2 (unknown).

a) Obtain the unbiased estimator for σ^2

b) Obtain the C-R lower bound for the unbiased estimator of σ^2

c) Does the variance of the UMVUE of σ^2 attain the CRLB

QUESTION FIVE (20 MARKS)

a) A random sample $X_1, X_2, ..., X_n$ comes from a binomial distribution iid as

$$p(x/\theta) = \begin{cases} \binom{n}{x} \theta^x (1-\theta)^{n-x} & x = 0, 1, 2, ..., n \\ 0 & elsewhere \end{cases}$$

Where n is known but θ is unknown $0 < \theta < 1$

i. Find a sufficient statistic for θ

ii. Find the UMVU estimator for θ^2.

iii. Find the UMVU estimator of $\theta(1-\theta)$

b) Let X_1 and X_2 be a sample of size 2 from a population X with mean μ and variable δ^2.

Two estimators for μ are proposed to be

$$\overline{\mu_1} = \frac{x_1 + x_2}{2} \qquad \overline{\mu_2} = \frac{x_1 + 2x_2}{3}$$

Which is the better estimator?

c) It is known that a certain proportion, say p, of manufactured parts is defective. From a supply of parts, n are choosen at random and tested. Define the readings (sample $X_1, X_2, ..., X_n$) to be 1 if good and 0 if defective. The, a good estimator

$$\hat{p} = 1 - \bar{X} = 1 - \frac{1}{n}(x_1 + x_2 + ... + x_n)$$

(i) Is $\hat{p}$ unbiased?

(ii) Is $\hat{p}$ consistent?

(iii) Show that $\hat{p}$ is an MLE of P.