(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

FIRST YEAR FIRST SEMESTER

SPECIAL/SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF MASTER OF SCIENCE IN STATISTICS

COURSE CODE: **STA 814**

COURSE TITLE: **STATISTICAL INFERENCE**

DATE: 03/10/18 **TIME:** 8 AM -11 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 3 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

SECTION A: COMPULSORY

QUESTION ONE (30MKS)

a) Let $x_1, x_2, \ldots, x_n$ be a random sample of size n from a population with Poisson distribution with parameter λ. Find the UMVUE of $e^{-\lambda}$

b) The following data gives the distribution of life (t)in hours of 337 electric bulbs of a certain manufacturing company. Assuming that the distribution of t follows the exponential law

$$f(t;\theta) = \begin{cases} \frac{1}{\theta} e^{\frac{-t}{\theta}} & \theta > 0, t \geq 0 \\ 0 & elsewhere \end{cases}$$

Find the estimate of the characteristic θ by the method of MLE given the data:

Life in hours	0 – 50	51 - 100	101 – 150	151 - 200	201- 250	251 - 300	301 - 350	351 - 400
frequency	100	68	48	31	42	21	15	12

c) i) Define the term sufficient statistics

ii) Let $x_1, x_2, \ldots, x_n$be a random sample from a negative exponential density

$$f(x;\theta) = \begin{cases} \theta e^{-\theta x} & x \geq 0 \\ 0 & elsewhere \end{cases}$$

estimate θ by the method of moments.

SECTION B: ANSWER ANY TWO QUESTIONS

QUESTION TWO (20 MKS)

Let $x_1, x_2, \ldots, x_n$ be a random sample from a normal distribution for which the mean is θ and variance σ^2 both parameters unknown.

a) Obtain the unbiased estimator for θ and σ^2

b) Obtain the C - R lower bound for θ and σ^2

QUESTION THREE (20 MKS)

a) Define the term an unbiased estimator

b) A random sample of size n families each of two children, gave n_1 families with zero boys, n_2 families with one boy and $n_3$families with two boys. It is assumed that the sexes of the two children are independent and that the probability of a boy is π at each birth. Find the MLE of $\pi$and show that it is an unbiased estimator.

QUESTION FOUR (20 MKS)

A random sample $x_1, x_2, \ldots, x_n$ comes from a binomial distribution iid as

$$P(x|\theta) = \begin{cases} \binom{n}{x} \theta^x 1 - \theta)^{n-x} & x = 0, 1, 2, \ldots, n \\ 0 & elsewhere \end{cases}$$

where n is known but θ is unknown $0 < \theta < 1$.

a) Find a sufficient statistic for θ

b) Find UMVU estimator of θ^2

c) Find UMVU estimator of $\theta(1 - \theta)$

QUESTION FIVE (20 MKS)

a) Define the term exponential family

b) Let the random sample $x_1, x_2, \ldots, x_n$ have a uniform density given by

$$f(x;\theta) = \frac{I(x)}{[\theta - \frac{1}{2}, \theta + \frac{1}{2}]} \qquad \text{with } -\infty < x < \infty$$

Obtain a sufficient statistic for θ hence or otherwise compute the MLE for θ.

c) Let $x_1, x_2, \ldots, x_n$ be a random sample from a uniform distribution on$(\mu - \sqrt{3}\,\delta,\ \mu + \sqrt{3}\,\delta)$, where μ and δ are unknown. Using the method of moments to obtain the estimators of $\mu$and δ.