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(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2016/2017 ACADEMIC YEAR

FIRST YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF MASTER OF SCIENCE IN STATISTICS

COURSE CODE: STA 805

COURSE TITLE: MULTIVARIATE ANALYSIS

DATE: 15/09/17 **TIME:** 8 AM -11 PM

INSTRUCTIONS TO CANDIDATES

Answer Any THREE Questions

TIME: 3 Hours

Question 1 (30 Marks)

(a.)Let $\underline{X}$ be a PX1 random vector with $V(\underline{Y}) = \Sigma$ and $E(\underline{Y}) = \underline{\mu}$.

Show that

(i). The matrix Σ is symmetric positive definite if the component random variables of $\underline{Y}$ are linearly independent (7 marks)

(ii). for a quadratic form

$Q(\underline{X}) = \underline{X}^1 A\underline{X}$,

$E(Q(\underline{X})) = \text{trace}(A\Sigma) + \underline{\mu}^1 A\underline{\mu}$

if A is a symmetric positive definitive matrix. (5 marks)

(b). Let $\underline{Y}$ be a PX1 r.vector and $\underline{X}$ be a P×1 random vector such that

$Cov(Y,X) = \Sigma_{yx}$

Let A be a kxpmatrix of constants and B be a pxq matrix of constants. Show that

$Cov(A\underline{Y},\underline{B}X) = A\Sigma_{yx}B^T$ (5 marks)

(c).(i). Define the Hotteling T^2 – statistic.

(ii). Show that the statistic T^2 is invariant of non – singular linear transformation. (8 marks) (iii). State the Roy's intersection principle for testing multivariate hypotheses. (5 marks)

(d). Let $\underline{Y}$ be a PX1 r.vector and $\underline{X}$ be a P×1 random vector such that

$Cov(Y,X) = \Sigma_{yx}$

Let A be a kxp matrix of constants and B be a pxq matrix of constants. Show that

$Cov(A\underline{Y},\underline{B}X) = A\Sigma_{yx}B^T$ (5 marks)

Question 2 (20 Marks)

(a). Show that if $\underline{Y}$ is $N_p[\underline{\mu}, \Sigma]$ then an rx1 subvector of $\underline{Y}$ has an r-variate normal distribution with the same means, variance and covariances as in the original p-variate normal distribution.

(6 marks)

(b). Using part (a) , or otherwise, show that any individual variable Y_i in $\underline{Y}$ is distributed as $N[\mu_i, \sigma_{ii},]$ where $\mu_i = E(Y_i)$

$\sigma_{ii} = V(Y_i)$ (6 marks)

(c). Show that if $\underline{Y}$ and $\underline{X}$ are jointly multivariate normal with $\Sigma_{yx} \neq 0$, then the conditional distribution of $\underline{Y}$ given as $\underline{X}$, $f(\underline{Y}/\underline{X})$ is multivariate normal with mean vector and covariance matrix.

$E(\underline{Y}/\underline{X}) = \underline{\mu}_y + \sum_{yx} \sum_{xx}^{-1}(\underline{X} - \underline{\mu}_x)$

$V(\underline{Y}/\underline{X}) = \sum_{yy} - \sum_{yx} \sum_{xx}^{-1} \sum_{yx}$

respectively. (8 marks)

Question 3 (20 Marks)

Let the random vector $\underline{V}$ be $N_4[\underline{\mu}, \sum]$ where $\underline{\mu} = (2\ 5\ -2\ 1)^1$,

and

$$\sum = \begin{pmatrix} 9 & 0 & 3 & 3 \\ 0 & 1 & -1 & 2 \\ 3 & -1 & 6 & -3 \\ 3 & 2 & -1 & 7 \end{pmatrix}$$

Let $\underline{V} = (Y_1\ Y_{2v}X_1\ X_2)^1$ (Hint: let entry at row 3, column 4 be -1)

(6 marks)

(a). Determine the distribution for $(Y_1\ Y_2)^1 / (X_1\ X_2)^1$

(b). Hence or otherwise, calculate

(i). $Cov(X_1, X_2)$

(ii). $Cov((X_1\ X_2)^1 / (Y_1\ Y_2)^1)$ (8 marks)

(c). If now $\underline{V} = (Y\ X_1\ X_2\ X_3)^1$, find the distribution of $Y / (X_1\ X_2\ X_3)^1$

Question 4 (20 Marks)

(a). Let $f(\underline{x}) = \frac{1}{(2\pi)^{p/2} |\Sigma|^{\frac{1}{2}}} exp - (\underline{X} - \underline{\mu})^1 \frac{\Sigma^{-1}}{2} (\underline{X} - \underline{\mu})$

where $\underline{X}$ is a px1 vector and $\sum$ is a positive definite matrix

Show that

(i). $f(\underline{x}) \geq 0$

(ii). $\int_{-\infty}^{\infty} f(\underline{x})\, d\underline{x} = 1$

where the integral denotes the P definite multiple integrals each over the interval $(-\infty, \infty)$. (8 marks)

(b). Let $\underline{Y}$ be distributed as $N_p[\underline{\mu}, \Sigma]$

Show that the moment generating function of $\underline{Y}$ is

$M_y(t) = \exp \underline{t}^1 \underline{\mu} + \underline{t}^1 \frac{\Sigma}{2} \underline{t}$ where $\Sigma = Var(\underline{Y})$ (6 marks)

(c). Using part (b) or otherwise, show that the moment generating function of $\underline{Z} = \underline{Y} - \underline{M}$

$M_z(t) = \exp \underline{t}^1 \frac{\Sigma}{2} \underline{t}$

(6 marks)

Question 5 (20 Marks)

(a). Show that $\underline{X}$ is $N_p[\underline{\mu}, \Sigma]$ *iff* $\underline{a}^1 X$ is $N_1[\underline{a}^1 \underline{\mu}, \underline{a}^1 \Sigma \underline{a}]$ (6 marks)

(b). (i). Briefly explain the importance of principal components analysis in statistics. (4 marks)

(ii). Let $\underline{X}_1 \sim N[\underline{\mu}_1, \Sigma]$ and $\underline{X}_2$ - $N[\underline{\mu}_2, \Sigma]$ denote random vectors with corresponding multivariate normal distribution. Assume $\underline{X}_1$ and $\underline{X}_2$ are independent. Assuming $\sum$ is unknown consider testing

$H_0 ; \underline{\mu}_1 = \underline{\mu}_2$

versus $H_1 ; \underline{\mu}_1$ different from $\underline{\mu}_2$

Determine a test for the hypothesis basing on the principal components of $\underline{X}_1$ and $\underline{X}_2$. (10 marks)