(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2016/2017 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: **STA 454**

COURSE TITLE: LARGE SAMPLE THEORY

DATE: 21/09/17 **TIME:** 8 AM -10 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE(30 MARKS)

1. (a) Name and explain any two probability tools for establishing consistency and asymptotic normality of estimators (4 mks)

   (b) The number of hits on a website follows a Poisson distribution, with a mean of 27 hits per hour. Find the probability that there will be 90 or more hits in three hours. (6 mks)

   (c) Suppose $a_n = \frac{1}{n} + \frac{3}{n^2} + \frac{1}{n^3}$ and $b_n = \frac{1}{n}$. Show that:

      i. $a_n \sim b_n$ (5 mks)

      ii. If $\{a_n\}$ and $\{b_n\}$ are symptotically equivalent then their relative error tends to zero. (5 mks)

   (d) In an election, a proportion p will vote for candidate G and a proportion $1 - p$ will vote for candidate B. In an election poll, a number of voters are asked for whom they will vote. Let X_i be the indicator random variable for the event "the i^{th} person interviewed will vote for G." A model for the election poll is that the the people to be interviewed are selected in such a way that the indicator random variables $X_1, X_2, \dots,$ are independent and have a Ber(p) distribution.

      i. Suppose $\bar{X_n}$ is used to predict p According to Chebyshev's inequality, how large should n be such that the probability that $\bar{X_n}$ is within 0.2 of the "true" p ia at least 0.9? (5 mks)

      ii. If $p > \frac{1}{2}$ candidate G wins; if $\bar{X_n} > \frac{1}{2}$ you predict that G will win. Find an n (as small as you can) such that the probability that you predict correctly is at least 0.9, if in fact $p = 0.6$ (5 mks)

QUESTION TWO (20 MARKS)

2. (a) Let $X_1, X_2, ..., X_n$ be observations that are i.i.d. Show that the sample mean is a consistent estimator for the population mean (5 mks)

   (b) Let a sequence $a_n = n$ and $b_n = 2n$. Show that a sequence $a_n = O(b_n)$ (5 mks)

   (c) If $X_1, X_2, ..., X_n$ is a sequence of random variables and if mean μ_n and standard deviation σ_n of X_n exists for all n and $\sigma_n \to -\infty$ as $n \to \infty$. Show that $x_n - \mu_n \xrightarrow{P} 0$ as $n \to \infty$ (5 mks)

   (d) The mean age of third and fourth year students at kibabii university is 22.3 years, and the standard deviation is 4 years. A random sample of 64 students is drawn. What is the probability that the average age of these students is greater than 23 years. (5 mks)

QUESTION THREE (20 MARKS)

3. (a) Calculate $P(|X - \mu| < a\sigma)$ exactly for $a = 1, 2, 3, 4$, when Y has an Exp(1) distribution and compare this with the bounds from Chebyshev's inequality. (6 mks)

   (b) For Geometric distribution $P(x) = 2^{-x}$; such that $x = 1, 2, ...$ prove that Chebyshev's inequality gives $P[|x - 2| \leq 2] > \frac{1}{2}$ while the actual probability is $\frac{15}{16}$ (7 mks)

   (c) Use the Chebyshev inequality to determine how many times a balanced coin must be tossed in order that the probability will be at least 0.80 that the ratio of the observed number of heads to the number of tosses will be between 0.4 and 0.6 (7 mks)

QUESTION FOUR (20 MARKS)

4. (a) Let X denote the number of flaws in a 1 inch length of copper wire. The probability mass function of X is presented in the following table.

X	0	1	2	3
$P(X = x)$	0.01	0.12	0.48	0.39

One hundred wires are sampled from this population. What is the probability that the average number of flaws per wire in this sample is less than 0.5? (6 mks)

(b) Let X_j, $j = 1, \ldots, n$ be i.i.d random variables such that $E[X_j] = \mu$, $\sigma^2(X_j) = \sigma^2$, both finite. Show that $E(X_n - X)^2 \overset{n\to\infty}{\longrightarrow} 0$ (4 mks)

(c) Suppose X is a random variable with mean μ and variance σ^2, then for any positive number a

$$P\{|X - \mu| \geq a\sigma\} \leq \frac{1}{a^2}$$

(10 mks)

QUESTION FIVE (20 MARKS)

5. (a) Show that if $F_n \to F$, then the convergence is uniform;

$$\sup_x |F_n| - F(x) \to 0$$

as $n \to \infty$ (10 mks)

(b) A dice is thrown 9000 times and a throw of 3 or 4 is observed 3240 times. Show that the dice cannot be regarded as an unbiased one and find the limits between which the probability of a throw of 3 or 4 lies. (10 mks)