



(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2016/2017 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: STA 448

COURSE TITLE: STOCHASTIC PROCESS II

DATE: 19/09/17

TIME: 3 PM -5 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 4 Printed Pages. Please Turn Over.

QUESTION 1: (30 Marks) (COMPULSORY)

- a) Let X have the distribution of the geometric distribution of the form
 $\text{Prob}(X = k) = p_k = q^k p, \quad k = 0, 1, 2, \dots$
Obtain the probability generating function and hence find its mean and variance
[9mks]
- b) Given that random variable X have probability density function $\text{pr}(X = k) = p_k, \quad k = 0, 1, 2, 3, \dots$ with probability generating function $P(S) = \sum_{i=1}^{\infty} p_k s^k$ and $q_k = p_k(X = k) = p_{k+1} + p_{k+2} + p_{k+3} + \dots$ with generating function $\phi(s) = \sum_{i=1}^{\infty} q_k s^k$
Show that $(1 - s)\phi(s) = 1 - p(s)$ and that $E(X) = \phi(1)$
[6mks]
- c) Find the generating function for the sequence $\{0, 0, 0, 5, 5, 5, 5, \dots\}$
[2mks]
- d) Define the following terms
- i. Absorbing state [1mk]
 - ii. Irreducible markov chains [1mk]
 - iii. Period of a state of markov chains [1mk]
- e) Classify the state of the following stochastic markov chain

$$\begin{array}{c} E_1 \quad E_2 \quad E_3 \\ E_1 \begin{bmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix} \\ E_2 \\ E_3 \end{array}$$

[10mks]

QUESTION 2: (20 Marks)

The difference – differential equation for pure birth process are

$$P'_n(t) = \lambda_n p_n(t) + \lambda_{n-1} p_{n-1}(t), \quad n \geq 1 \text{ and}$$

$$P'_0(t) = -\lambda_0 p_0(t), \quad n = 0.$$

Obtain $P_n(t)$ for a non – stationary pure birth process (Poisson process) with $\lambda_n = \lambda$ given that

$$P_0(t) = \begin{cases} 1 & \text{for } n = 0 \\ 0 & \text{otherwise} \end{cases}$$

Hence obtain its mean and variance

QUESTION 3: (20 Marks)

a) Let X have a Poisson distribution with parameter λ i.e.

$$\text{Prob}(X = k) = p_k = \frac{e^{-\lambda} \lambda^k}{k!}, \quad k = 0, 1, 2, 3, \dots$$

Obtain the probability generating function of X and hence obtain its mean and variance [5mks]

b) Using Feller's method, find the mean and variance of the difference – differential equation

$$P'_n(t) = -n(\lambda + \mu)p_n(t) + (n-1)\lambda p_{n-1}(t) + \mu(n+1)p_{n+1}(t), \quad n \geq 1 \text{ given}$$

$$m_1(t) = \sum_{n=0}^{\infty} n p_n(t), \quad m_2(t) = \sum_{n=0}^{\infty} n^2 p_n(t) \text{ and}$$

$$m_3(t) = \sum_{n=0}^{\infty} n^3 p_n(t) \text{ conditioned on } p_1(0) = 0, \quad p_n(0) = 0, \quad n \neq 0 \quad \text{[14mks]}$$

