(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: **STA 448**

COURSE TITLE: **STOCHASTIC PROCESS II**

DATE: **09/10/18** **TIME:** **3 PM -5 PM**

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION 1: (30 Marks) (COMPULSORY)

a) Define the following terms

i. Transient state **[1mk]**

ii. Ergodic state **[1mk]**

iii. Recurrent state **[1mk]**

b) Let $\boldsymbol{X}$ have a binomial distribution with parameter $\boldsymbol{n}$ and $\boldsymbol{p}$ i.e.

$$\boldsymbol{Prob}\ (X = k) = p_k = \binom{n}{k} p^k q^{n-k}, \quad k = 0, 1, 2, 3, \ldots, n$$

Obtain the probability generating function of $\boldsymbol{X}$ and hence find its mean and variance. **[9mks]**

c) Given that random variable $\boldsymbol{X}$ have probability density function $pr(X = k) = p_k \quad k = 0, 1, 2, 3, \ldots$ with probability generating function $P(S) = \sum_{i=1}^{\infty} p_k s^k$ and $q_k = p_k(X = k) = p_{k+1} + p_{k+2} + p_{k+3} + \cdots$ with generating function $\phi(s) = \sum_{i=1}^{\infty} q_k s^k$

Show that $(1 - s)\phi(s) = 1 - p(s)$ and that $E(X) = \phi(1)$ **[6mks]**

d) Find the generating function for the sequence $a_k = \left\{ \begin{matrix} n \\ k \end{matrix} \right\}$ for fixed $\boldsymbol{n}$ **[2mks]**

e) Classify the state of the following transitional matrix of the markov chains

$$\begin{array}{c} \\ E_1 \\ E_2 \\ E_3 \\ \vdots \\ \vdots \end{array} \begin{array}{c} \begin{array}{cccccc} E_1 & E_2 & E_3 & E_4 & E_5 & \cdots \end{array} \\ \begin{bmatrix} 1/2 & 1/2 & 0 & 0 & 0 & \cdots \\ 1/2 & 0 & 1/2 & 0 & 0 & \cdots \\ 1/2 & 0 & 0 & 1/2 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1/2 & 0 & 0 & 0 & 0 & \cdots \end{bmatrix} \end{array}$$

[10mks]

QUESTION 2: (20 Marks)

a) Let X have the distribution of the geometric distribution of the form $Prob\ (X = k) = p_k = q^{k-1}\, p\ ,\quad k = 1, 2, 3\ ...$
Obtain the probability generating function and hence find its mean and variance **[6mks]**

b) The difference – differential equation for pure birth process are
$P'_n(t) = \lambda_n p_n(t) + \lambda_{n-1} p_{n-1}(t),\quad n \geq 1$ and
$P'_0(t) = -\lambda_0 p_0(t),\ n = 0.$
Obtain $P_n(t)$ for a non – stationary pure birth process (Poisson process) with $\lambda_n = \lambda$ given that

$$P_0(t) = \begin{cases} 1 & for\ n = 0 \\ 0 & otherwise \end{cases}$$

Hence obtain its mean and variance **[14mks]**

QUESTION 3: (20 Marks)

a) Let X have a Poisson distribution with parameter λ i.e.
$Prob\ (X = k) = p_k = \frac{e^{-\lambda}\lambda^k}{k!},\ \ k = 0, 1, 2, 3, ...$
Obtain the probability generating function of X and hence obtain its mean and variance **[6mks]**

b) Using Feller's method, find the mean and variance of the difference – differential equation
$P'_n(t) = -n(\lambda + \mu)p_n(t) + (n - 1)\lambda p_{n-1}(t) + \mu(n + 1)p_{n+1}(t),$
$n \geq 1$ given
$m_1(t) = \sum_{n=0}^{\infty} n p_n(t)\ ,\ m_2(t) = \sum_{n=0}^{\infty} n^2 p_n(t)$ and
$m_3(t) = \sum_{n=0}^{\infty} n^3 p_n(t)$ conditioned on $p_1(0) = 0,\ p_n(0) = 0,\ n \neq 0$
[14mks]

QUESTION 4: (20 Marks)

a) Define the following terms

  i. Absorbing state **[1mk]**

  ii. Markov chains **[1mk]**

  iii. Cycling or a periodic **[1mk]**

b) Classify the state of the following stochastic markov chain

$$\begin{array}{c} \\ E_1 \\ E_2 \\ E_3 \end{array} \begin{array}{c} \begin{array}{ccc} E_1 & E_2 & E_3 \end{array} \\ \begin{bmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix} \end{array}$$

[17mks]

QUESTION 5: (20 Marks)

The difference – differential equation for the simple birth – death process are
$P'_n(t) = -n(\lambda + \mu)p_n(t) + (n-1)\lambda p_{n-1}(t) + (n+1)\mu p_{n+1}(t),\ n \geq 1$ and
$P'_0(t) = \mu p_1(t),\ n = 0$
Obtain $P_n(t)$ for a simple Birth – Death process with $\lambda_n = n\lambda$ and $\mu_n = n\mu$ given that $P_n(0) = \begin{cases} 1 & for\ n = 1 \\ 0 & for\ n = 0 \end{cases}$