(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

THIRD YEAR SECOND SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: **STA 346**

COURSE TITLE: QUALITY CONTROL AND ACCEPTANCE SAMPLING

DATE: 18/10/18 **TIME:** 3 PM -5 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION 1: (30 Marks) (COMPULSORY)

a) Give the three main objectives of a control chart **[3mks]**

b) What are the main applications of a control chart **[4mks]**

c) A large batch of items to be inspected using a single sampling scheme specified by the following values $n = 40,\ c = 2,\ \theta_1 = 0.02$, and $\theta_2 = 0.1$

   i. Define the operating characteristic of this sampling plan **[2mks]**

   ii. Find the probability of accepting a lot of quality $\theta = 0.05$ **[3mks]**

   iii. Find the consumer's risk and the producer's risk **[4mks]**

d) Workout the O.C curve and the ARL function for S^2-chart with upper warning limits given by $P[\sum(x_i - \bar{x})^2 > k] \leq 0.05$ and action is taken only if two consecutive values of S^2 fall beyond the upper warning limit (take $n = 12,\ \theta = \frac{\sigma^{2*}}{\sigma}$ and $\theta \rightarrow (-\infty, 0, \infty)$ **[6mks]**

e) suppose that the mean has shifted from μ to μ^* but σ^2 remain unchanged assuming normality (take $\alpha = 0.002$)

   i. Find the probability that the process is under control for the $\bar{x} - chart$ **[4mks]**

   ii. Show that the Average Run Length function of the $\bar{x} - chart$ is given by $\frac{1}{1-P(\theta)}$. Assuming that samples taken from the process are independent, where θ is the incoming quality **[4mks]**

QUESTION 2: (20 Marks)

a) Explain each of the following concepts

   i. Average sampling numbers [ASN] **[2mks]**

   ii. Average outgoing quality [AOQ] **[3mks]**

   iii. Lot tolerance percent defective (LTPD) **[3mks]**

   iv. Acceptance Quality Level [AQL] **[2mks]**

b) What do you understand by the moving average chart? Explain clearly how you can use it to determine whether a system is out of control or not. **[5mks]**

c) When do we use S^2- chart? Explain clearly how you can use it to determine whether a system is out of control or not. If $n = 4$ and $\alpha = 0.02$, obtain its upper action and warning limits. **[5mks]**

QUESTION 3: (20 Marks)

a) If n is large and p is moderately small and we let $\lambda = np$, obtain C-chart for the number of defectives per unit. (Take $\alpha = 0.001$ for action limit and $\alpha = 0.025$ for warning limit **[7mks]**

b) Obtain a single sampling for the proportion of defectives, fixing the producer's risk $\alpha = 0.09$ at $\theta_1 = 0.05$ and the consumer's risk $\beta = 0.1$ at $\theta_1 = 0.1$ and hence give your conclusion **[6mks]**

c) A large batch of items is to be tested by using double sampling inspection scheme specified by the following numbers $n_1 = 20,\ n_2 = 40,\ c_1 = 0,\ c_2 = c_3 = 2$

   i. Obtain an expression for the probability of accepting a batch in which the true proportion of defective is θ **[4mks]**

   ii. Obtain the value of this probability when $\theta = 0.05$ and $\theta = 0.1$ **[3mks]**

QUESTION 4: (20 Marks)

a) Briefly describe the important steps in constructing an $\bar{x} - chart$ **[5mks]**

b) Briefly compare the single sampling plan and the double sampling plan **[3mks]**

c) The data below are samples means and sample ranges for ten consecutive samples, each sample consisting of five measurements of a continuous random variable x. Assuming x is normally distributed plot $\bar{x} - control\ chart$ and comment on the degree of control

Sample No.	1	2	3	4	5	6	7	8	9	10
Sample mean	126.2	127.4	126.6	129.8	126.0	125.0	126.8	132.0	127.4	126.2
Sample Range	8	6	7	6	8	7	6	19	6	7

$a_n = 0.4299\ for\ n = 5$ **[6mks]**

d) Explain briefly how you use control chart for fractional defective (p –chart) to determine whether the process is in control or not and hence show its warning and action limit on a p-chart. (take $\alpha = 0.002$ for action limit and $\alpha = 0.05$ for warning limit) **[6mks]**

QUESTION 5: (20 Marks)

a) i. Construction a sequential sampling plan from a Bernoulli population that following values $\theta_0 = 0.02$, $\theta_1 = 0.08$, $\alpha = 0.05$ and $\beta = 0.1$ **[3mks]**

ii. An inspector test 40 units from a large lot. Would he have come to a decision to reject or accept the lot if he found the 10^{th}, 18^{th}, and 23^{rd} unit defective. **[3mks]**

b) A company purchases large lots of items using a single sampling plan for which $n = 4$ and $c = 0$

i. Find the probability of acceptance of a lot in terms of proportion of defective items it contains. **[2mks]**

ii. What is the probability of

1. A lot containing 50% defective being accepted **[2mks]**
2. A lot containing 10% defective being rejected **[2mks]**

iii. Estimate the AQL (θ) corresponding to a producers risk of 5% and LTPD (θ) corresponding to consumer's risk of 10% **[3mks]**

iv. If rectification is agreed on, find the expression for the average outgoing quality (AOQ) in terms of the incoming quality. Find AOQ if $\theta = 0.05$ **[2mks]**

v. Calculate the average total inspection (ATI) of lots of size 100 of quality $\theta = 0.05$ **[3mks]**