(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2016/2017 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER

SPECIAL/SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

(MATHEMATICS)

COURSE CODE: STA 443

COURSE TITLE: PROBABILITY AND MEASURE

DATE: 15/09/17 **TIME:** 3 PM -5 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

1. (a) i. What are Lebesgue measurable sets? (2 mks)

   ii. Describe any two Lebesgue measurable sets. (4 mks)

   (b) Show that if A and B are subsets of E such that $\mu^*(A)$ and $\mu^*(B)$ are both finite then, $|\mu^*(A) - \mu^*(B)| \leq \mu^*(A\Delta B)$ where $(A\Delta B) := (AB^c) \cup (BA^c)$ (5 mks)

   (c) If μ is a σ-finite measure on an algebra A of subsets of S. Show that:

   i. there exists an increasing sequence and (4 mks)

   ii. there exists a disjoint σ-finite sequence. (4 mks)

   (d) If $A \subset B$, show that $\mu^*(A) \leq \mu^*(B)$. (3 mks)

   (e) Prove that if $0 \leq f_n \to f$ almost everywhere and $\int f_n d\mu \leq A < \infty$, then f is integrable and $\int f d\mu \leq A$ (3 mks)

   (f) State and briefly explain any two types of measures on the intervals over the real line. (5 mks)

QUESTION TWO (20 MARKS)

2. (a) Let $\{E_i \subset R^n : i \in N\}$ is countable collection of R^n. Show that

$$\mu^*(\cup_i^\infty E_i) \le \sum_{i=1}^{\infty} \mu^*(E_i)$$

(5 mks)

(b) Let $f_{XY}(x, y) = \frac{1}{50}(x^2 + y^2)$ if $0 < x < 2, 1 < y < 4$ and zero otherwise. Find $P(X + Y > 4)$ (5 mks)

(c) Suppose A and B are independent events in the sample space. Show that A^c and B are independent. (5 mks)

(d) Prove that every monotone function is measurable. (5 mks)

QUESTION THREE (20 MARKS)

3. (a) Find the integral $f(x, y) = x^2 + y^2$, on the domain

$$D = \left\{(x, y) \in R^2 : 0 < x < 1, x^2 < y < x\right\}$$

(8 mks)

(b) Suppose $f = \sum_i x_i I_{Ai}$ is a non-negative simple function, $\{A_i\}$ being decomposition of S into F sets, show that

$$\int f d\mu = \sum_i x_i \mu(A_i)$$

(6 mks)

(c) Let $r, s, t \in [1, \infty]$ satisfy $\frac{1}{r} + \frac{1}{s} = \frac{1}{t}$. Prove that for all measurable f and g defined on a space (X, A, μ), given $||fg||_t \le ||f||_r ||g||_s$ (6 mks)

QUESTION FOUR (20 MARKS)

4. (a) State and explain two properties of conditional expectation (4 mks)

   (b) Find the mathematical expectation of a random variable with (9 mks)

      i. uniform distribution over the interval $[a, b]$
      ii. triangle distribution
      iii. exponential distribution

   (c) Show that if $\{f_n\}$ is a sequence of non-negative measurable functions, and $\{f_n(x) : n \leq 1\}$ increases monotonically to $f(x)$ for each x then

$$\lim_{n \to \infty} \int_E f_n(x)dm = \int_E f dm$$

(7 mks)

QUESTION FIVE (20 MARKS)

5. (a) State Fubini's theorem (2 mks)

   (b) Let f_1 and f_2 be measurable functions on a common domain. Show that each set $\{\omega : f_1(\omega) < f_2(\omega)\}$, $\{\omega : f_1(\omega) = f_2(\omega)\}$ and $\{\omega : f_1(\omega) > f_2(\omega)\}$ is measurable (8 mks)

   (c) Suppose $\{B_n\}$ is sequence of independent events and $\sum_n Pr\{B_n\} = \infty$. Show the probability that B_n occurs infinitely often is one. (10 mks)