(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: **STA 442**

COURSE TITLE: **MULTIVARIATE ANALYSIS**

DATE: 12/10/18 **TIME:** 11.30 AM -1.30 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

(a) Define the following terms

- (i) Random vector (1mks)
- (ii) Positive definite matrix (2mks)

(b) Let $\underline{x} = [5,1,3]$ and $\underline{y} = [-1,3,1]$. Find

- (i) The length of $\underline{x}$ (2mk)
- (ii) The angle between $\underline{x}$ and $\underline{y}$ (3mks)
- (iii) The length of the projection of $\underline{x}$ on $\underline{y}$ (1mk)

(c) Let $A = \begin{bmatrix} 9 & -2 \\ -2 & 6 \end{bmatrix}$

- (i) Is A symmetric? Give reason (1mk)
- (ii) Obtain Eigen value (3mks)
- (iii) Show that A is positive definite (6mks)

(d) Consider the following $n = 3$ observations on $p = 2$ variables

Variable 1: $x_{11} = 2, x_{21} = 3, x_{31} = 4$

Variable 1: $x_{12} = 1, x_{22} = 2, x_{32} = 4$

- (i) Compute the sample means $\bar{x}_1$ and $\bar{x}_2$ and the sample variances S_{11} and S_{22} (4mks)
- (ii) Compute the sample covariance S_{12} and the sample correlation coefficient r_{12} and interpret these quantities (5mks)
- (iii) Display the sample mean array $\bar{x}$, the sample correlation array R and the sample variance-covariance S_{12} (4mks)

QUESTION TWO (20MARKS)

(a) Let $\underline{x}$ be a p-variate random vector with mean vector $\underline{\mu}$ and variance covariance matrix Σ, show that $E(\underline{XX'}) = \Sigma + \underline{\mu\mu}'$, hence show that $E(\underline{X}'A\underline{X}) = trace(A\Sigma) + \underline{\mu}'A\underline{\mu}$ where A is a symmetric matrix of constants. (6mks)

(b) Let $\underline{x}$ be a trivariate random vector such that

$E(\underline{x}) = 0$ and $var(\underline{x}) = \begin{bmatrix} 5 & 2 & 3 \\ 2 & 3 & 0 \\ 3 & 0 & 2 \end{bmatrix}$. Find the expected value of the quadratic form

$$Q = (x_1 - x_2)^2 + (x_2 - x_3)^2 + (x_3 - x_1)^2$$

(4mks)

(c) Using the variance-covariance matrix in part (b) find

(i) The variance of $Y = x_1 - 2x_2 + x_3$ (2mks)

(ii) The variance covariance matrix of $Y = (Y_1, Y_2)$ where $Y_1 = x_1 + x_2$ and $Y_2 = x_1 + 2x_2 + x_3$ (4mks)

(d) Let $\underline{x}$ be a p-variate random vector and $\underline{a}p \times 1$ vector of constants, show that

$$E[(\underline{x} - \underline{a})(\underline{x} - \underline{a})' \ ] = [(E(\underline{x}) - \underline{a})(E(\underline{x} - \underline{a})'] + var(\ \underline{x}\ )$$ (4mks)

QUESTION THREE (20 MARKS)

(a) Assume $\underline{x}' = (x_1, x_2, x_3)$ is normally distributed with mean vector $\underline{\mu} = (1, -1,2)$ and variance matrix $\Sigma = \begin{bmatrix} 4 & 0 & -1 \\ 0 & 5 & 0 \\ -1 & 0 & 2 \end{bmatrix}$. Find the distribution of $3x_1 - 2x_2 + x_3$ (7mks)

(b) Show that the sample mean is an unbiased estimator of $\underline{\mu}$ and that the sample variance is biased estimator of matrix Σ (6mks)

(c) Find the maximum likelihood estimators of the mean vector $\underline{\mu}$ and covariance matrix Σ based on the data matrix

$$x = \begin{bmatrix} 3 & 6 \\ 4 & 4 \\ 5 & 7 \\ 4 & 7 \end{bmatrix}$$ (7mks)

QUESTION FOUR (20 MARKS)

(a) Given the data matrix $x = \begin{bmatrix} 1 & 9 & 10 \\ 4 & 12 & 16 \\ 2 & 10 & 12 \\ 5 & 8 & 13 \\ 3 & 11 & 14 \end{bmatrix}$

Define $X_c = X - 1\,\overline{x}\,'$ as the mean corrected data matrix.

(i) Obtain the mean corrected data matrix (5mks)

(ii) Obtain the sample covariance matrix and hence the generalized variance (8mks)

(iii) Verify that columns of mean corrected data matrixare linearly dependent. (3mks)

(iv) Specify a vector $a' = [a_1\ a_2 a_3]$ that establishes the linear dependence (4mks)

QUESTION FIVE (20 MARKS)

(a) Let $\underline{x}$ be a random vector having the covariance matrix

$$\Sigma = \begin{bmatrix} 25 & -2 & 4 \\ -2 & 4 & 1 \\ 4 & 1 & 9 \end{bmatrix}$$

(i) Obtain the population correlation matrix (ρ) and $V^{\frac{1}{2}}$ (6mks)

(ii) Multiply your matrices to check the relation$V^{\frac{1}{2}}\rho V^{\frac{1}{2}}$ (4mks)

(b) Given that$f(x_1 + x_2) = \begin{cases} x_1 x_2, & 0 \le x_1 \le x_2 \le 1 \\ 0 & e\backslash w \end{cases}$

Obtain $E(x_1/x_2)$ where $E[x]$ means expected value of$x$. (10 mks)