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(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2016/2017 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: **STA 441**

COURSE TITLE: TIME SERIES

DATE: 12/09/17 TIME: 11.30 AM -1.30 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION 1: (30 Marks) (COMPULSORY)

a) Explain the following terms as used in time series analysis:

i) Stationary process **(1mk)**
ii) Stationarity in the weak sense **(1mk)**
iii) Random walk process **(1mk)**
iv) Moving average process **(1mk)**
v) White noise process **(1mk)**

b) Find the autocovariance function ($\boldsymbol{\sigma(h)}$) and the autocorrelation function ($\boldsymbol{\rho(h)}$) of a moving average process of order q (MA(q)). **(8mks)**

c) Consider autoregressive process of order 1 (AR(1)) given by
$X_t = \propto X_{t-1} + e_t$, where $\propto$ is a constant.

i) If $|\propto| < 1$, show that X_t may be expressed as infinite order of a MA process. **(4mks)**

ii) Find its autocovariance function $\left(\sigma(h)\right)$ and its autocorrelation function $\left(\rho(h)\right)$. **(3mks)**

d) Transform a time series $\{X_t\}$ into another series $\{Y_t\}$ where
$Y_t = \sum_{j=-\infty}^{\infty} a_j X_{t-j}$ and $X_t = e^{i\lambda t}$
and state the changes in its amplitude, wavelength and phase angle. **(5mks)**

e) Find the spectral density function of an AR(1) process given by **(5mks)**
$X_t = \propto X_{t-1} + e_t$, where $|\propto| < 1$

QUESTION 2: (20 Marks)

a) Suppose we have data up to time $n(x_1, x_2, \ldots, x_n)$

i) Show that minimum mean squared error forecast of x_{n+k} is the conditional mean of x_{n+k} at time n.
i.e. $\hat{x}(n,k) = E(x_{n+k}/x_1, x_2, \ldots, x_n)$ **(6mks)**

ii) Consider the AR(1) model $X_t = \propto X_{t-1} + e_t$, $|\propto| < 1$.
Forecast x_{n+3}. **(2mks)**

b) Transform a moving average filter $\{X_t\}$ into another series $\{Y_t\}$ by the linear operator given that

$X_t = e^{i\lambda t}$ and $Y_t = \sum_{j=-\infty}^{\infty} a_j X_{t-j}$

Where

$$a_j = \begin{cases} \frac{1}{2m+1}, & j = 0, \mp 1, \mp 2, \ldots\ldots, \mp m \\ 0, & otherwise \end{cases}$$ **(12mks)**

QUESTION 3: (20 Marks)

a) Consider an AR(1) process with mean μ given by
$X_t - \mu = \alpha(X_{t-1} - \mu) + e_t, t = 1, 2, 3, \ldots\ldots$

Find the estimates of the parameters $\propto$ and μ using the method of least squares. **(8mks)**

b) Consider a second order process AR(2) given by
$X_t = {}^1/_3 X_{t-1} + {}^2/_9 X_{t-2} + e_t$.
Show that this process is stationary and hence obtain its ACF **(12mks)**

QUESTION 4: (20 Marks)

a) i) Briefly describe the main objectives in the analysis of a time series. **(3mks)**

ii) State the unique feature that distinguishes time series from other branches of statistics. **(1mk)**

iii) Identify the main stages in setting up a Box-Jenkins forecasting model. **(4mks)**

b) Show that the AR(2) process given $\boldsymbol{X_t = X_{t-1} - {}^1/_2 X_{t-2} + e_t}$ is stationary and hence find its ACF. **(12mks)**

QUESTION 5: (20 Marks)

a) If an observed values $(X_1, X_2, \ldots, X_n)$ on a discrete time series forms $n-1$ pairs of observation $(X_1, X_2), (X_2, X_3), \ldots, (X_{n-1}, X_n)$ regarding the first observation in each pair as one variable and second observation as a second variable

Find:

i) The correlation coefficient r_1 between X_t and X_{t-1} **(5mks)**

ii) The correlation between observations at a distance k apart. **(2mk)**

b) The data below gives the average quarterly prices of a commodity for four (4) years.

Year	I	II	III	IV
1997	50.4	40.8	47.5	49.8
1998	38.3	33.6	53.2	69.5
1999	67.2	53.2	60.7	42.6
2000	55.1	56.4	61.6	65.1

Calculate the seasonal indices. **(6mks)**

c) Consider a moving average process given by $\boldsymbol{X_t = e_t + \beta e_{t-1}}$, where $(\boldsymbol{\beta_0 = 1, \beta_1 = 1})$.

Find its spectral density function. **(7mks)**