



(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: STA 441

COURSE TITLE: TIME SERIES

DATE: 02/10/18

TIME: 11.30 AM -1.30 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 4 Printed Pages. Please Turn Over.

QUESTION 1: [30 Marks] (COMPULSORY)

a) Explain the following terms as used in time series analysis:

- i) Stationary process [1mk]
- ii) Stationarity in the strong sense [1mk]
- iii) Random walk process [1mk]
- iv) Autoregressive process [1mk]
- v) Purely random process [1mk]

b) Transform a time series $\{X_t\}$ into another series $\{Y_t\}$ where

$$Y_t = \sum_{j=-\infty}^{\infty} a_j X_{t-j} \text{ and } X_t = e^{i\lambda t}$$

and state the changes in its amplitude, wavelength and phase angle.

[8mks]

c) Consider autoregressive process of order 1 (AR(1)) given by

$$X_t = \alpha X_{t-1} + e_t, \text{ where } \alpha \text{ is a constant.}$$

- i) If $|\alpha| < 1$, show that X_t may be expressed as infinite order of a MA process. [4mks]
- ii) Find its autocovariance function $(\sigma(h))$ and its autocorrelation function $(\rho(h))$. [3mks]

d) Show that the AR(2) process given $X_t = X_{t-1} - \frac{1}{2} X_{t-2} + e_t$ is stationary and hence find its ACF.

[10mks]

QUESTION 2: [20 Marks]

a) Suppose we have data up to time $n(x_1, x_2, \dots, x_n)$

- i) Show that minimum mean squared error forecast of x_{n+k} is the conditional mean of x_{n+k} at time n .

$$\text{i.e. } \hat{x}(n, k) = E(x_{n+k} / x_1, x_2, \dots, x_n)$$

[6mks]

- ii) Consider the AR(1) model $X_t = \alpha X_{t-1} + e_t$, $|\alpha| < 1$.

Forecast x_{n+3} .

[2mks]

b) Consider a second order process AR(2) given by

$$X_t = \frac{1}{3} X_{t-1} + \frac{2}{9} X_{t-2} + e_t.$$

Show that this process is stationary and hence obtain its ACF

[12mks]

QUESTION 3: [20 Marks]

- a) Find the autocovariance function ($\sigma(h)$) and the autocorrelation function ($\rho(h)$) of a moving average process of order q (MA(q)). [8mks]

- b) Transform a moving average filter $\{X_t\}$ into another series $\{Y_t\}$ by the linear operator given that

$$X_t = e^{i\lambda t} \text{ and } Y_t = \sum_{j=-\infty}^{\infty} a_j X_{t-j}$$

Where

$$a_j = \begin{cases} \frac{1}{2m+1}, & j = 0, \pm 1, \pm 2, \dots, \pm m \\ 0, & \text{otherwise} \end{cases}$$

[12mks]

QUESTION 4: [20 Marks]

- a) i) Briefly describe the main objectives in the analysis of a time series.

[3mks]

- ii) State the unique feature that distinguishes time series from other branches of statistics.

[1mk]

- iii) Identify the main stages in setting up a Box-Jenkins forecasting model.

[4mks]

- b) Consider an AR(1) process with mean μ given by

$$X_t - \mu = \alpha(X_{t-1} - \mu) + e_t, t = 1, 2, 3, \dots$$

Find the estimates of the parameters α and μ using the method of least squares.

[6mks]

- e) Find the spectral density function of an AR(1) process given by

$$X_t = \alpha X_{t-1} + e_t, \text{ where } |\alpha| < 1$$

[6mks]

QUESTION 5: [20 Marks]

- a) If an observed values (X_1, X_2, \dots, X_n) on a discrete time series forms $n - 1$ pairs of observation $(X_1, X_2), (X_2, X_3), \dots, (X_{n-1}, X_n)$ regarding the first observation in each pair as one variable and second observation as a second variable

Find:

- i) The correlation coefficient r_1 between X_t and X_{t-1} [5mks]
- ii) The correlation between observations at a distance k apart. [2mks]

- b) The data below gives the average quarterly prices of a commodity for four (4) years.

Year	I	II	III	IV
1997	50.4	40.8	47.5	49.8
1998	38.3	33.6	53.2	69.5
1999	67.2	53.2	60.7	42.6
2000	55.1	56.4	61.6	65.1

Calculate the seasonal indices. [6mks]

- c) Consider a moving average process given by $X_t = e_t + \beta e_{t-1}$, where $(\beta_0 = 1, \beta_1 = 1)$.

Find its spectral density function. [7mks]