



(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

(MATHEMATICS)

COURSE CODE: STA 441

COURSE TITLE: TIME SERIES ANALYSIS

DATE: 18/12/17

TIME: 3 PM -5 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 4 Printed Pages. Please Turn Over.

QUESTION 1: (30 Marks) (COMPULSORY)

a) Explain the following terms as used in time series analysis:

- i) Stationary process (1mk)
- ii) Stationarity in the weak sense (1mk)
- iii) Moving average process (1mk)
- iv) Autoregressive process (1mk)
- v) White noise process (1mk)

b) Find the autocovariance function ($\sigma(h)$) and the autocorrelation function ($\rho(h)$) of a moving average process of order q (MA(q)). (8mks)

c) Consider autoregressive process of order 1 (AR(1)) given by
 $X_t = \alpha X_{t-1} + e_t$, where α is a constant.

- i) If $|\alpha| < 1$, show that X_t may be expressed as infinite order of a MA process. (4mks)
- ii) Find its autocovariance function ($\sigma(h)$) and its autocorrelation function ($\rho(h)$). (3mks)

d) Transform a time series $\{X_t\}$ into another series $\{Y_t\}$ where

$$Y_t = \sum_{j=-\infty}^{\infty} a_j X_{t-j} \text{ and } X_t = e^{i\lambda t}$$

and state the changes in its amplitude, wavelength and phase angle.

(5mks)

e) Find the spectral density function of an AR(1) process given by
 $X_t = \alpha X_{t-1} + e_t$, where $|\alpha| < 1$

(5mks)

QUESTION 2: (20 Marks)

- a) Suppose we have data up to time $n(x_1, x_2, \dots, x_n)$
- i) Show that minimum mean squared error forecast of x_{n+k} is the conditional mean of x_{n+k} at time n .
i.e. $\hat{x}(n, k) = E(x_{n+k}/x_1, x_2, \dots, x_n)$ (6mks)
- ii) Consider the AR(1) model $X_t = \alpha X_{t-1} + e_t$, $|\alpha| < 1$.
Forecast x_{n+3} . (2mks)
- b) Transform a moving average filter $\{X_t\}$ into another series $\{Y_t\}$ by the linear operator given that
 $X_t = e^{i\lambda t}$ and $Y_t = \sum_{j=-\infty}^{\infty} a_j X_{t-j}$
Where
$$a_j = \begin{cases} \frac{1}{2m+1}, & j = 0, \pm 1, \pm 2, \dots, \pm m \\ 0, & \text{otherwise} \end{cases}$$
 (12mks)

QUESTION 3: (20 Marks)

- a) Consider an AR(1) process with mean μ given by
 $X_t - \mu = \alpha(X_{t-1} - \mu) + e_t, t = 1, 2, 3, \dots$
Find the estimates of the parameters α and μ using the method of least squares. (8mks)
- b) Consider a second order process AR(2) given by
 $X_t = \frac{1}{3} X_{t-1} + \frac{2}{9} X_{t-2} + e_t$.
Show that this process is stationary and hence obtain its ACF (12mks)

QUESTION 4: (20 Marks)

- a) i) Briefly describe the main objectives in the analysis of a time series. (3mks)
ii) State the unique feature that distinguishes time series from other branches of statistics. (1mk)
iii) Identify the main stages in setting up a Box-Jenkins forecasting model. (4mks)
- b) Show that the AR(2) process given $X_t = X_{t-1} - \frac{1}{2}X_{t-2} + e_t$ is stationary and hence find its ACF. (12mks)

QUESTION 5: (20 Marks)

- a) If an observed values (X_1, X_2, \dots, X_n) on a discrete time series forms $n - 1$ pairs of observation $(X_1, X_2), (X_2, X_3), \dots, (X_{n-1}, X_n)$ regarding the first observation in each pair as one variable and second observation as a second variable
Find:
i) The correlation coefficient r_1 between X_t and X_{t-1} (5mks)
ii) The correlation between observations at a distance k apart. (2mks)
- b) The data below gives the average quarterly prices of a commodity for four (4) years.

Year	I	II	III	IV
1997	50.4	40.8	47.5	49.8
1998	38.3	33.6	53.2	69.5
1999	67.2	53.2	60.7	42.6
2000	55.1	56.4	61.6	65.1

Calculate the seasonal indices. (6mks)

- c) Consider a moving average process given by $X_t = e_t + \beta e_{t-1}$, where $(\beta_0 = 1, \beta_1 = 1)$.
Find its spectral density function. (7mks)