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(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2016/2017 ACADEMIC YEAR

SECOND YEAR FIRST SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: **STA 241**

COURSE TITLE: **STATISTICS AND PROBABILITY**

DATE: 11/09/17 **TIME:** 3 PM -5 PM

INSTRUCTIONS TO CANDIDATES
Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE 30 (MKS) COMPULSORY

a) State 4 characteristics of a binomial distribution (4 marks)

b) The discrete random variable Y had the following probability distribution: (Y can only assume values o, 1,2 and 3).

y	o	1	2	3
P(y)	2a	3a	7a	6a

i) Find the value of c. (2 marks)

ii) Find the mean and variance of Y. (4 marks)

c) Suppose that Y has density function

$$f(y) = \begin{cases} cy, 0 \leq y \leq 2, \\ 0, \text{elsewhere} \end{cases}$$

i) Find the value of C that makes *f(y)* a probability density function (2 marks)

ii) Find *F(y)* (1 mark)

d) The achievement scores for a college entrance examination are normally distributed with mean 75 and standard deviation 10. What fraction of the scores lies between 80 and 90? (4 marks)

e) Let Y be a random variable with moment-generating function $m(t) = \frac{1}{6}e^{t} + \frac{2}{6}e^{2t} + \frac{3}{6}e^{3t}$.

Find (i) E(Y) (4 marks)

(ii) Var(Y) (4 marks)

(iii) The distribution of Y (2 marks)

f) Four-week summer totals in section of the Midwest United States have approximately a gamma distribution with $\alpha = 4.8$ and $\beta = 6.0$.Find the mean and variance of the four-week rainfall totals (2marks)

g) Let $X_1$and X_2have joint density function given as

$$f(x_1, x_2) = \begin{cases} 3x_1, \ 0 \leq x_1 \leq 1 \\ 0, \ \text{elsewhere} \end{cases}$$

Find the Mean and variance ofX_1 (4marks)

QUESTION 2 (20 MARKS)

a) A random variable Y follows a binomial distribution with n trials and probability of success p, show that

(i) The moment-generating function for Y is m(t) = $(pe^t + q)^n$, (2 marks)

(ii) E$[Y]$ = np (2 marks)

(iii) $Var[Y]$= npq, where q= 1- p (4 marks)

b) A large shipment of books contains 5% with imperfect bindings. Use the Poisson approximation of the binomial distribution to compute the probability that among 100 books:

(i) At most 3 will have imperfect bindings (3 marks)
(ii) Exactly 5 will have imperfect bindings (3 marks)
(iii) At least 4 will have imperfect bindings (3 marks)

c) Customers arrive at a checkout counter in a department store according to a Poisson distribution at an average of seven per hour. During a given hour, what is the probability that at least two customers arrive? (3 marks)

QUESTION 3 (20 MARKS)

a) The table below shows the probability distribution of a random variable Y

y	0	1	3	4
P(y)	0.125	0.25	0.375	0.25

Compute the standard deviation of Y (5 marks)

b) A committee of 4 people is to be selected at random from among 10 people, of whom 3 are women and 7 are men. If Y denotes the number of women selected, determine the probability distribution of Y. (6 marks)

c) The time in minutes taken by a milk man to deliver milk to a nearby dairy is normally distributed with a mean of 12 minutes and a variance of 4 minutes. Estimate the number of days during the year when he takes

i) Longer than 17 min (3 marks)

ii) Less than 10 min (3 marks)

iii) Between 9 min and 13 min (3 marks)

QUESTION 4 (20 MARKS

Let Y be a random variable with probability density function

$$F(y) = \begin{cases} k(2-y), & 0 \le y \le 2 \\ 0, & elsewhere \end{cases}$$

Where k is a positive constant.

a) Find k such that *f (y)* is a valid probability density function. (3 marks)

b) Find the cumulative distribution function for Y. Make sure to specify *F (y)* for all real values of y. (5 marks)

c) Sketch the graphs of *f(y)* and *F(y)* (2 marks)

d) Find E(Y) and V(Y) (8 marks)

e) The standard deviation (2 marks)

QUESTION 5 (20 MARKS)

a) Suppose that 35% of the items produced by a factory are defective. If 20 of the items are inspected, what is the probability that :

i)the number of defective items are exactly 12 (3 marks)

ii) More than 4 items are defective (4 marks)

iii) the number of defective items lie between 12 and 15 inclusive (4marks) iv)Find the mean and variance of the number of defective items. (2 marks)

b) The distance between flaws on a long cable is exponentially distributed with mean 2m. Find:

i) the probability that the distance between two flows is greater than 15m (3 marks)

iii) The probability that the distance between two flaws is between 8 and 20(4 marks)