(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

SECOND YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: **STA 241**

COURSE TITLE: **STATISTICS AND PROBABILITY**

DATE: 17/01/18 **TIME:** 9 AM -11 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE [30 MARKS]

a) Distinguish between a discrete and continuous random variables giving an example in each case (4mks)

b) A continuous random variable X has probability distribution

$$f(x) = \begin{cases} \frac{2x}{5} + 2k & 0 \leq x \leq 3 \\ 0 & \text{Otherwise} \end{cases}$$

Determine the value of the constant k and calculate the probability $p(1 \leq x \leq 2)$. (6mks)

c) Let X be a random variable with density function given as

$$f(x) = \begin{cases} \theta\, e^{-\theta x} & x \geq 0 \\ 0 & \text{Otherwise} \end{cases}$$

Find the moment generating function for X and hence find the mean and variance for X. (9mks)

d) If X has a Poisson distribution with

$$p(x = 2) = \frac{2}{3}\, p(x = 1)$$

Find $p(x = 0)$ (4mks)

e) A discrete random variable has the probability distribution

x	0	1	2	3
$f(x)$	$\frac{84}{286}$	$\frac{44}{286}$	$\frac{54}{286}$	$\frac{4}{286}$

Calculate

i. $\mu_x = E(X)$ (2mks)

ii. $E(X^2)$ (2mks)

iii. $Var(X)$ (1mk)

f) If X and Y are independent random variable and $Z = X + Y$. Show that moment generating function of Z is given by

$M_Z(t) = M_X(t)M_Y(t)$ (2mks)

QUESTION TWO [20 MARKS]

a) A geometric random variable X has the probability distribution

$$f(x) = \begin{cases} p(1-p)^{x-1} & x = 1,2,3, \dots \dots \dots \dots \dots \\ 0 & \text{Otherwise} \end{cases}$$

i. Show that the moment generating function is given by

$M_X(t) = \frac{pe^t}{1-(1-p)e^t}$ (5mks)

ii. Use $M_X(t)$ to find the mean and variance of X. (4mks)

b) In a sample of 1000 students, the mean of a certain test is 14 and standard deviation is 2.5. Assuming the distribution to be normal, find

(i) How many students scored between 12 and 15 (4mks)

(ii) How many scored above 18 (2mks)

(iii) How many scored below 8 (2mks)

(iv) The value of k such that $p(X < k) = 0.8686$ (3mks)

QUESTION THREE [20 MARKS]

a) discrete random variable X has probability mass function given by

$$f(x) = \begin{cases} \left(\frac{1}{3}\right)^x \left(\frac{2}{3}\right) & x = 0,1,2,\ldots\ldots \\ 0 & \text{elsewhere} \end{cases}$$

Determine the factorial moment generating function of X and hence compute the mean and variance (11mks)

b) The local authorities in Nairobi city install 1000 electric in the city. If these lamps have average life of 1000 burning hours with a standard deviation of 200 hours assuming normality.

(i) What number of lumps might be expecting to fail in the first 800 burning hours? (3mks)

(ii) After what period of burning hours would you expect that 10 % would fail? (3mks)

c) Suppose that the probability of female birth is 0.3. If 10 individual are selected in this population. What is the probability of getting 6 women? (3mks)

QUESTION FOUR [20 MARKS]

a) A random variable Y has probability density function given as

$$f(x) = \begin{cases} m + ny^2\,, & 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

If its mean is $\frac{2}{3}$, determine the value of m and n . (6mks)

b) 280 watches are inspected and the numbers of defective per set are recorded.

Number of defective	Number of set
0	20
1	100
2	72
3	35
4	43
5	10

Estimate the average number of defective per set and expected frequency of 0, 1,2,3,4 and 5. Assuming Poisson distribution and fit a Poisson distribution (10mks)

c) Show that $Var(X+Y) = Var(X) + Var(Y)$ if X and Y are independent random variables. (4mks)

QUESTION FIVE [20 MARKS]

a) A random variable X has a poison distribution such that $3p(x=1) = p(x=2)$. Find $p(x \leq 3)$ (6mks)

b) Prove that the mean and the variance of a poison distribution are:-

$\mu = \lambda$ and $\sigma^2 = \lambda$ (8mks)

c) Assume a random variable $X \sim B(100,\ 0.35)$the standard deviation of the population is given as $\sigma = 4.77$

Find

(i) Probability of failure (q)

(ii) $p(x \leq 45)$

(iii) $p(25 \leq x \leq 45)$ (6mks)