65

(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE (INFORMATION TECHNOLOGY)

COURSE CODE: STA 225

COURSE TITLE: PROBABILITY AND STATISTICS II

DATE: 31/07/18 **TIME:** 2 PM -4 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 4 Printed Pages. Please Turn Over.

QUESTION ONE [30MARKS]

a.) Differentiate between a discrete random variable and a continuous random variable. Give one example in each case. [4marks]

b.) Let X be a random variable with a Poisson distribution f(x) = $\begin{cases} e^{-\lambda}\lambda^{x}, x = 0,1,2,\dots. \\ o,\ elsewhere \end{cases}$

where λ a constant.Show is that f (x) is a discrete P.d.f. [4marks]

c.) A continuous random variable X has probability distribution given by

f(x) = $\begin{cases} \frac{6x-4}{15} & 0 \leq x \leq 3 \\ o,\ otherwise \end{cases}$

i.) Show that f(x) is a continuous P.d.f. [4marks]

ii.) Calculate the probability P ($1 \leq X \leq 2$) [3marks]

d.) Consider the P.d.f given by f(x) = $\begin{cases} \frac{1}{18}(3+2x), 2 \leq x \leq 4 \\ o,\ elsewhere \end{cases}$. Find:

i.) E(x) [3marks]

ii.) E(x^2) and hence var(x) [5marks]

e.) Let X be a random variable with density function given as f(x)= $\begin{cases} \theta e^{-\theta x}, x \geq 0 \\ 0, otherwise \end{cases}$. Find:

i.) the M.g.f. of X [4marks]

ii.) the mean of X [3marks]

QUESTION TWO [20MARKS]

a.) A company that makes sweets has a label weight 20.4grams of each sweet. Assume that the distribution of the weight of these sweet is normal with mean 21.37grams and standard deviation of 0.4grams.Let X denote the weight of a single sweet selected at random from the production .Find: i.) P($x \geq 22.07$) [3marks]

ii.) The number of sweets with weight less than 20.4grams as indicated in the label in a packet of 2,500 sweets. [4marks]

b.) If X is a random variable with probability density function

f(x)= $\begin{cases} \left(\frac{32}{63}\right)\frac{1}{2^{x}}, x = 0,1,2,3,4,5 \\ o,\ otherwise \end{cases}$, Find: E(x) [3marks]

ii.) Var(x) [3marks]

c.) A random variable X has a Poisson distribution such that $3P(X=1) = P(X=2)$.

Find: $P(X \leq 3)$ [7marks]

QUESTION THREE [20MARKS]

a.) Define the moment generating function of a random variable X where it exists. [2marks]

b.) i.) Show that the moment generating function of a binomial distribution is given by

$$M_x(t) = (Pe^t + q)^n$$ [6marks]

ii.) Hence find the mean and variance of X. [6marks]

c.) The Bungoma county government has installed 100 electric lamps in Bungoma town. The lamps have an average lifetime of 1000 burning hours with standard deviation 200 hours assuming normality.

i.) Find the numbers of lamps that are expected to fail in the first 800 hours of burning. [3marks]

ii.) Find after how many hours of burning would 10% of the lamps be expected to fail. [3marks]

QUESTION FOUR [20MARKS]

a.) A random variable X has a probability density function given by

$f(x) = \begin{cases} (m + nx^2), & 0 < x < 1 \\ 0, & otherwise \end{cases}$. Given that $E(X) = \frac{2}{3}$, determine the value of m and n. [6marks]

b.) In a firm that manufactures watches, 280 watches are inspected and the numbers of defective per set are recorded.

Number of defective	Number of sets
0	20
1	100
2	72
3	35
4	43
5	10

i.) Estimate the average number of defective per set and expected frequency of 0, 1,2,3,4 and5. [2marks]

ii.) Fit a Poisson distribution of the data. [8marks]

QUESTION FIVE [20MARKS]

a.) The random variable Z has the probability distribution as shown in the table below

Z	2	3	5	7	11
P(z)	1/6	1/3	1/4	a	B

If E [Z] = $\frac{14}{3}$, find: i.)the value of a and b. [6marks]

ii.)the variance of Z [5marks]

b.) $f(x) = \begin{cases} \left(\frac{1}{3}\right)^x \left(\frac{2}{3}\right), & x = 0,1,2, \ldots \ldots \\ o, & elsewhere \end{cases}$. Determine the factorial moment generating function of X. Hence compute the mean. [9marks]