(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2016/2017 ACADEMIC YEAR

SECOND YEAR FIRST SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: **STA 211**

COURSE TITLE: STATISTICS AND PROBABILITY

DATE: 28/09/17 **TIME:** 8 AM -10 AM

INSTRUCTIONS TO CANDIDATES
Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 5 Printed Pages. Please Turn Over.

Question 1 Compulsory (30 marks)

a) The number of cars arriving at a parking garage during morning rush hour is believed to follow a Poisson distribution with a rate of 1 per minute. Let Y denote the number of cars arriving during a 20 minute rush hour period.

i) Find the expected value of Y ($E[Y] = \lambda$) (1 mark)

ii) Suppose that the garage has 25 empty parking sports at 6:00 a.m. If no cars leave the garage during the period, what is the probability that a car arriving immediately after 6:20 will get a parking spot. (2 marks)

b) The discrete random variable Y had the following probability distribution: (Y can only assume values o, 1,2 and 3).

y	o	1	2	3
P(y)	a	3a	5a	7a

i) Determine the value of a. (2 marks)

ii) Find the mean and variance of Y. (4 marks)

iii) Find p(0< y < 3) (2 marks)

c) Suppose that Y has density function

$$f(y) = \begin{cases} 2e^{-2y}, & y > 0, \\ 0, & y \leq 0 \end{cases}$$

i) Find P(1≤ y ≤ 3)(2 marks)

ii) Find P(y > 0.5)(1 mark)

d) Let Y be a random variable with moment-generating function m(t) = $\frac{1}{6}e^{t} + \frac{2}{6}e^{2t} + \frac{3}{6}e^{3t}$.

Find (i) E(Y) (3 marks)

(ii) V(Y) (4 marks)

(iii) The distribution of Y(3 marks)

e) Six-week summer totals in section of the Midwest United States have approximately a gamma distribution with $\alpha = 3.2$ and $\beta = 4.0$.Find the mean and variance of the four-week rainfall totals (2marks)

f) Sate four characteristics of a binomial distribution (4 marks)

Question 2 (20 marks)

a) The amount of coffee (in tonnes) that a given company sells in a day is found to be a numerically valued random phenomenon with a pdf specified as follows:

$$f(y)=\begin{cases} Ky, 0 \leq y \leq 3 \\ K(5+y), 3 \leq y \leq 5 \\ 0, \quad elsewhere \end{cases}$$

Find

i) the value of k that makes this a probability density function (3 marks)

iii) the probability that the number of coffee sold on a certain day would be more than 3 tonnes(3 marks)

iv) the probability that the number of coffee sold on a certain day would be less than 3 tonnes (3 marks)

v) the probability that the number of coffee sold on a certain day would be between 2 and 4tonnes(3 marks)

b) Show that if Y has a gamma distribution with parameters α and β, then μ = E(Y) = $\alpha\beta$ and $\sigma^2 = V(Y) = \alpha\beta^2$. (8 marks)

Question 3 (20 marks)

a) A random variable Y follows a binomial distribution with n trials and probability of success p, show that

(i) $E[Y] = np$ (4 marks)

iii) $V[Y] = npq$, where $q = 1 - p$ (5 marks)

b) A large shipment of books contains 5% with imperfect bindings. Use the Poisson approximation of the binomial distribution to compute the probability that among 100 books:

(i) At most 2 will have imperfect bindings (3 marks)

(ii) Exactly 5 will have imperfect bindings (4 marks)

(iii) At least 4 will have imperfect bindings (4 marks)

Question 3 (20 marks)

a) The table below shows the probability distribution of a random variable Y

y	0	1	3	4
P(y)	0.125	0.25	0.375	0.25

Compute the mean and standard deviation (5 marks)

b) A committee of 4 people is to be selected at random from among 10 people, of whom 3 are women and 7 are men. If Y denotes the number of women selected, determine the probability distribution of Y. (6 marks)

c) The time in minutes taken by a milk man to deliver milk to a nearby dairy is normally distributed with a mean of 12 minutes and a variance of 4 minutes. Estimate the number of days during the year when he takes

i) Longer than 17 min (3 marks)

ii) Less than 10 min (3 marks)

iii) Between 9 min and 13 min (3 marks)

Question 4 (20 marks

Let x_1 and x_2 have joint density function given by

$$f(y)=\begin{cases} c(x_1{}^2 + x_2), & x_1 = -1, 0, 1: x_2 = 1,2 \\ 0, & elsewhere \end{cases}$$

Find (a) the constant c. (2 marks)

b) the marginal probability function of $x_1$and x_2(6 marks)

c) the conditional probability of x_2 given $x_1 = 0$(6 marks)

d)the conditional mean and variance $x_2$given $x_1 = 0$ (6 marks)

Question 5 (20 marks)

a) Suppose that 35% of the items produced by a factory are defective. If 20 of the items are inspected, what is the probability that :

i)the number of defective items are exactly 12 (2 marks)

ii) More than 4 items are defective (4 marks)

iii) the number of defective items lie between 12 and 15 inclusive (4marks)

vi) Find the mean and variance of the number of defective items. (2 marks)

b) Let Y be a continuous random variable with probability density function given by

$$f(y) = \begin{cases} y, & 0 < y < 1 \\ 2 - y, & 1 < y < 2 \\ 0, & \text{elsewhere} \end{cases}$$

Calculate the mean and variance of Y (8 marks)