(*Knowledge for Development*)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

SECOND YEAR FIRST SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: **STA 210**

COURSE TITLE: **PROBABILITY AND STATISTICS**

DATE: 18/10/18 **TIME: 8 AM -10 AM**

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of **4 Printed Pages**. Please Turn Over.

QUESTION ONE [30 MARKS]

a. Let $f(x)=\begin{Bmatrix}\frac{2}{5}-\frac{2x}{225}, 0\le x\le 15\\ 0, eleswhere\end{Bmatrix}$,

(i)show that $f(x)$ is a P.d.f [2MARK]

(ii) Find the expected value of X [3MARKS]

(iii) Find the variance of X [4MARKS]

b. Let the probability density function of X be given by

$f(x)=\begin{Bmatrix}k(\frac{1}{3})^x, x=1,2,3,---\\ 0, eleswhere.\end{Bmatrix}$

(i) Find the value of k. [4MARKS]

(ii) Find the moment generating function of X. [6MARKS]

(iii) Using the M.g.f. find the mean of X. [4MARKS]

c. (i) Let X be bin(2,p) and let Y be bin(4,p).If $\Pr(X\ge 1)=\frac{5}{9}$, find $\Pr(Y\ge 1)$

[5MARKS]

(ii)The probability that an experiment will succeed is 0.8.If the experiment is repeated until four successful outcomes occur, what is the expected number of repetitions required?

[2MARKS]

QUESTION TWO [20 MARKS]

a. The probability distribution of a discrete random variable X is given by

$f(x)=k\binom{3}{x}\binom{4}{3-x}, x=0,1,2,3.$ where k is a constant.

$=0, eleswhere.$

Show that $k=\frac{1}{35}$. [3MARKS]

Hence find the (i) Variance of X. [8MARKS]

(ii) The cumulative density function of X. [4MARKS]

b. Let $M_x(t) = \left[\frac{1}{3} + \frac{2}{3}e^t\right]^5$ be the M.g.f. of a arandom variable X.

(i)Write the probability density function of X [2MARK]

(ii) Find Pr(X=2or3) [3MARKS]

QUESTION THREE [20MARKS]

a. The random variable X has a probability density function given by

$$f(x) = \frac{1}{\sqrt{2\pi}} e \frac{(x-1)^2}{2}, -\infty \leq x \leq \infty$$
$$= 0, eleswhere.$$

Show that the moment generating function of X, $M_x(t)$ is given by $M_{x(t)} = e^{t+\frac{t^2}{2}}$ [6MARKS]

b. Let $\psi(t) = \ln[m(t)]$,where $m(t)$ is the moment – generating function of a Normal distribution. Prove that $\psi^I(0) = \mu$ and $\psi^{II}(0) = \sigma^2$. [6MARKS]

c. Suppose that the PH of soil samples taken from a certain region is normally distributed with mean PH 6.00 and standard deviation PH0.1.If the PH of a randomly selected soil sample from this region is determined what is the probability that the :

(i).resulting PH is between 5.90 and 6.15? [3MARKS]

(ii) resulting PH is atmost 5.95 [2MARKS]

d. Let X be a random variable and C any real number .Prove that $Var(CX) = C^2Var(X)$.

[3MARKS]

QUESTION FOUR[20 MARKS]

a. Define the probability density function f(x) for a Binomial random variable X,with parameters n and p. [2MARKS]

b. If X is a binomial random variable with parameters n and p ,obtain the moment generating function of X. Hence obtain the mean and variance of X. [12MARKS]

c. If X is N(3,16),then find:(i) $P(4 \leq x \leq 8)$ [3MARKS]

(ii) $P(-2 \leq x \leq 1)$ [3MARKS]

QUESTION 5[20 MARKS]

a. Let X and Y be independent and identically distributed Poisson random variables,that is

$$f(x)=\begin{cases}\frac{e^{-\lambda}\lambda^{x}}{x!}, x=0,1,2,3,---\\ 0, eleswhere\end{cases} \text{ and } f(y)=\begin{cases}\frac{e^{-\lambda}\lambda^{y}}{y!}, y=0,1,2,3,---\\ 0, eleswherw\end{cases}$$

Let Z = X+ Y, (i) Find theM.g.f. of Z . [6MARKS]

ii. Hence evaluate E(Z) and Var(Z) [6MARKS]

b. A random variable X has Poisson distribution such that $3P(X=1)=P(X=2)$.

.Find $P(X\leq 2)$ [5MARKS]

c. The number of cars per minute passing a certain point on a road is Poisson distributed with mean 4.Find the probability that 8 cars pass in 2 minutes. [3MARKS]