(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

SECOND YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: **STA 210**

COURSE TITLE: **PROBABILITY AND STATISTICS**

DATE: 18/01/18 **TIME:** 2 PM -4 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE [30 MARKS]

a) Define a characteristic function of random variable X where it exist and state its two properties (4mks)

b) Let X be a continuous random variable with the following probability density function

$$f(x) = \begin{cases} x + \frac{1}{2}, & 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

(i) Verify that $f(x)$ is indeed a pdf (2mks)

(ii) Determine the mean of the distribution (2mks)

(iii) Determine the standard deviation (3mks)

c) 200 bulbs are inspected and the number of defective per set is recorded:

No. of defective	No. of sets
0	30
1	72
2	43
3	35
4	20

Estimate the average number of defective per set and expected frequency of 0,1,2,3 and 4.Assuming Poisson distribution and fit a Poisson distribution (8mks)

d) Let X and Y be random variables. If X and Y are not independent, then prove that

$Var(X + Y) = Var(X) + Var(Y) + 2\,Cov(X, Y)$ (4mks)

e) Let X be a discrete random variable taking possible values of 0 and 1 with probability $1-p$ and p respectively. Obtain moment generating function of X and compute the mean and variance (7mks)

Q UESTION TWO[20 MARKS]

a) A random variable X has a mean 3 and variance 4. Find $E(X^2 + 3X + 5)$ (4mks)

b) If X is a random variable with probability density function.

$$f(x) = \begin{cases} \left(\frac{32}{63}\right)\frac{1}{2^x} & x = 0,1,2,3,4,5 \\ 0 & \text{elsewhere} \end{cases}$$

Find $E\ (x)$ and $var\ (x)$ (5mks)

c) A Geological study has indicated that the probability of striking oil by oil prospect company in a certain region is 0.2. Find the probability that the first strike of oil comes after drilling three dry wells. (3mks)

d) A company that makes sweets have a label weight 20.4 grams of each sweet. Assume that the distribution of the weight of these sweet is normal with mean 21.37 grams and standard deviation of 0.4 grams. Let X denotes the weight of a single sweet selected at random from the production. Find

(i) $p(x \geq 22.07)$

(ii) . The Number of sweets with the weight less than 20.4 grams as indicated in the label in a Packet of 2,500 sweets. (5mks)

e) The mean of a binomial distribution is 27 and standard deviation 3. Calculate the values of n, p and q (3mks)

QUESTION THREE [20 MARKS]

a) Define the moment generating function of random variable X where it exists and state two of its properties. (4mks)

b) Show that the moment generating function of binomial distribution is given by

$M_X(t) = (Pe^t + q)^n$ (6mks)

Hence prove that $E(X) = np$ and $Var(X) = npq$ (6mks)

c) Find the probability that 7 of 10 persons will recover from a tropical diseases given that the probability is 0.80 that any one of them will recover from the disease (4mks)

QUESTION FOUR [20 MARKS]

a) Define factorial moment generating function (2mks)

b) The probability density function of Poisson distribution is given by

$$f(x) = \begin{cases} \frac{e^{-\lambda}\lambda^x}{x!} & x = 0,1,2, \dots.. \\ 0 & \text{elsewhere} \end{cases}$$

(i) Show that factorial moment generating function is given by

$$\phi_X(t) = e^{\lambda(t-1)}$$

(ii) Use $\phi_X(t)$ to determine the mean and variance (6mks)

c) A machine produces 1% defective items. Suppose it produces 1000 items. What is the probability that 4 items selected at random is defective (3mks)

d) If X has a Poisson distribution with

$$p(x = 1) = \frac{3}{2}\, p(x = 2). \text{ Find } p(x = 4)$$ (4mks)

QUESTION FIVE [20 MARKS]

a) A random variable has probability density function given by

$$f(x) = \begin{cases} \frac{1}{5}\left(\frac{4}{5}\right)^{x} & x = 0,1,2, \dots \dots \dots \\ 0 & \text{elsewhere} \end{cases}$$

(i) Prove that moment generating function of X is given by (5mks)

$$M_X(t) = \frac{\frac{1}{5}}{1 - \frac{4}{5}e^{t}}$$

(ii) Use $M_X(t)$ to determine mean and variance (5mks)

b) A committee of 4 people is to be selected at random among 10 people of whom 3 are women and 7 are men. Find the probability of having 2 women

(3mks)

c) Suppose that X is the height of an adult female human is normally distributed with mean of 160 and standard deviation is 10.

Calculate the probability that

(i) $p(X < 170)$ (2mks)

(ii) $p(170 < X \leq 175)$ (3mks)

(iii) $p(X > 180)$ (2mks)