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(Knowledge for Development)

KIBABII UNIVERSITY
UNIVERSITY EXAMINATIONS
2016/2017 ACADEMIC YEAR
FIRST YEAR SECOND SEMESTER
SPECIAL/ SUPPLEMENTARY EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE
MATHEMATICS

COURSE CODE: STA 142

COURSE TITLE: INTRODUCTION TO PROBABILITY

DATE: 28/09/17

TIME: 3 PM -5 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 4 Printed Pages. Please Turn Over.

QUESTION ONE (30 MARKS)

- a) Define the following terms as used in probability (4marks)
- Mutually exclusives events
 - Exhaustive events
 - Compound events
 - Equally likely events
- b) Find the number of distinct permutations that can be formed from the words
- PHYSIOLOGY
 - STATISTICS (4marks)
- c) State four different school of thought on the concept of probability (4marks)
- d) State two theorems of probability (2marks)
- e) Show that $p[A^c] = 1 - P[A]$ (2marks)
- f) In a class of 20 children, 4 of the 9 boys and 3 of the 11 girls are in the athletics. A person from the class is chosen to be in the egg and spoon race on sports day. Find the probability that the person chosen is a female or in the athletics team (4marks)
- g) The p.d.f. of discrete random variable Y is given by $P(Y = y) = Cy^2$ for $y=0, 1,2,3,4$. Given that C is a constant, find the value of c (3marks)
- h) A continuous random variable has p.d.f $f(x) = kx^2$ for $0 \leq x \leq 4$. Find the value of the constant k. Find also $p(1 \leq x \leq 3)$. (4marks)
- i) Suppose that a game is to be played with a single die. In this game, a player wins Ksh. 20 if a turns up and Ksh. 40 if a 4 turns up, he loses Ksh. 30 if a 6 turns up. Find the expected sum of money to be won. (3marks)

QUESTION TWO (20 MARKS)

- a) The discrete random variable X has probability function

$$P(X = x) = \begin{cases} \frac{kx}{(x^2 + 1)}, & x = 2, 3 \\ \frac{2kx}{(x^2 - 1)}, & x = 4, 5 \\ 0, & \text{otherwise} \end{cases}$$

- i) Find the value of k and write down the probability distribution of X (5marks)
- ii) Find the probability that X is less than 3 or greater than 4 (2marks)
- iii) Find $F(3.2)$ (2marks)
- iv) Find $E[x]$ and $\text{var}[x]$ (6marks)

b) Find the distribution function for the random variable of probability function

$$f(X = x) = \frac{1}{9}x^2, \text{ where } 0 \leq x \leq 3 \quad (5\text{marks})$$

QUESTION THREE (20 MARKS)

a) A random variable X has a probability density function

$$f(x) = Ax(6 - x)^2 \quad 0 \leq x \leq 6$$

$$= 0 \quad \text{elsewhere.}$$

- i. Find the value of the constant A (2marks)
- ii. Find the mean (3marks)
- iii. Find the variance (3marks)
- iv. Find the mode (3marks)
- v. Find the standard deviation (2marks)

b) X is a random variable with p.d.f as shown

$$f(X = x) = \frac{1}{8}x, \text{ where } 0 \leq x \leq 4$$

Find the median and inter-quartile range (7marks)

QUESTION FOUR (20 MARKS)

a. The events A and B are such that $p(A/B) = 0.4$, $p(A \cap B) = 0.25$ and $p(A \cup B) = 0.12$.

- i. Calculate the value of $p(B)$ (2marks)
- ii. Give a reason why A and B are not independent (2marks)
- iii. Calculate the value of $p(A \cap B')$ (3marks)

b. A group of girls at a school are entered for Advanced Level mathematics models. Each girl takes only module M_1 or module M_2 or both M_1 and M_2 . The probability that a girl is taking

M_2 given that she is taking M_1 is $1/5$. The probability that a girl is taking M_1 given that she is taking M_2 is $1/3$. Find the probability that

- i. A girl selected at random is taking both M_1 and M_2 (6marks)
 - ii. A girl selected at random is taking only M_1 (4marks)
- c. If two events A and B are independent show that A and B also independent (3marks)

QUESTION FIVE (20 MARKS)

a) State and prove Bayes' Theorem (7mark)

b) Three Urns of the same type have the following number of balls.

First urn: 2 black 1 white

Second urn: 1 black 2 white

Third urn: 2 black 2 white

One of the urns is selected and one ball is drawn. It turns out to be white. What is the probability of drawing a white ball again, the first one not having been returned? (4marks)

c) There are three alternative proposal before a businessman to start a new project.

- Proposal A: profit of Ksh 5M with a probability of 0.6 or a loss of ksh 80,000 with a probability of 0.4.
- Proposal B: profit of Ksh 10M with a probability of 0.4 or a loss of ksh 200,000 with a probability of 0.6.
- Proposal C: profit of Ksh 4.5M with a probability of 0.8 or a loss of ksh 50,000 with a probability of 0.2.

If he wants to maximize the profit and minimize the loss, which proposals should he prefer? (7marks)

d) State the difference between permutation and combination. (2marks)