230

(Knowledge for Development)

# KIBABII UNIVERSITY

## UNIVERSITY EXAMINATIONS

## 2017/2018 ACADEMIC YEAR

## FIRST YEAR SECOND SEMESTER

## MAIN EXAMINATION

## FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

**COURSE CODE:** **STA 142**

**COURSE TITLE:** INTRODUCTION TO PROBABILITY

**DATE:** 09/08/18 **TIME:** 9 AM -11 AM

---

**INSTRUCTIONS TO CANDIDATES**

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 6 Printed Pages. Please Turn Over.

**Question 1 Compulsory (30 marks)**

a) Distinguish between the following terms as used in probability

(i) Mutually exclusive events and independent events (2 marks)
(ii) Probability mass function and probability density function (2 marks)

b) Suppose that A and B are two events. Write expressions involving unions, intersections, and complements that describe the following:

(i) Both events occur.
(ii) At least one occurs.
(iii) Neither occurs.
(iv) Exactly one occurs. (4 marks)

c) Two events A and B are such that P(A) = 0.2, P(B) = 0.3, and $P(A \cup B) = 0.4$. Find the following;

(i) $P(A \cap B)$ (1 mark)
(ii) $P(A^c \cup B^c)$ (1 mark)
(iii) $P(A^c \cap B^c)$ (1 marks)
(iv) $P(A^c \vee B)$ (2 marks)

d) A fleet of nine taxis is to be dispatched to three airports in such a way that three go to airport A, five go to airport B, and one goes to airport C. In how many distinct ways can this be accomplished? (2 marks)

e) Prove that $\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$. (3 marks)

f) A group of four components is known to contain two defectives. An inspector tests the components one at a time until the two defectives are located. Once she locates the two defectives, she stops testing, but the second defective is tested to ensure accuracy. Let Y denote the number of the test on which the second defective is found.

(i) Find the probability distribution for Y. (3 marks)
(ii) Construct a probability histogram for p(y) (1 mark)

g) Let Y be a random variable with p(y) given in the accompanying table.

| y | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| p(y) | 0.4 | 0.3 | 0.2 | 0.1 |

Find (i) $\mu = E(Y)$ (ii) E(1/Y) (iii) $E(Y^2-1)$
(iv) Show that $Var(Y) = E(Y^2) - \mu^2$ and hence determine the variance . (5 marks)

h) Suppose that Y has density function

$$f(y) = \begin{cases} cy, & 0 \le y \le 2, \\ 0, & \text{elsewhere} \end{cases}$$

(i) Find the value of c that makes *f(y)* a probability density function.(2 marks)
(ii) Find *F(y)* (1 mark)

**Question 2 (20 marks)**

a) A business office orders paper supplies from one of three vendors, $v_1, v_2,$ or $v_3$. Orders are to be placed on two successive days, one order per day. Thus, $(v_2, v_3)$ might denote that vendor $v_2$ gets the order on the first day and $v_3$ gets the order on the second day.

i) List the sample points in this experiment of ordering paper on two successive days.

(3 marks)

ii) Let A denote the event that the same vendor gets both orders and B the event that $v_2$ gets at least one order.

iii) Find P(A), (P(B), (P($A \cup B$), P($A \cap B$) by summing the probabilities of the sample points in these events. (7 marks)

b) If A and B are independent events with P(A) = 0.5 and P( B ) = 0.2, find the following:

i) P ($A \cup B$)
ii) P ($A^c \cap B^c$)
iii) P ($A^c \cup B^c$) (4 marks)

c) If two events, A and B, are such that P(A) = 0.5, P(B) = 0.3, and P($A \cap B$) = 0.1, find the following:

(i) P (A|B)

(ii) P (B|A)

(iii) P (A|$A \cup B$)

(iv) P (A|$A \cap B$)

(v) P ($A \cap B \vee A \cup B$) (7 marks)

**Question 3 (20 marks)**

a) The discrete random variable Y the following probability distribution: (Y can only assume values 0, 1, 2 and 3).

| y | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| P(y) | $\frac{1}{3}$ | $\frac{1}{4}$ | $\frac{1}{5}$ | ? |

(i) What is the missing probability p(3)? (2 marks)

(ii) Find the mean and variance of Y. (5 marks)

b) A subcommittee of 6 members is to be formed out of a group of 7 men and 4 ladies. Calculate the probability that the committee will consist of :

i) Exactly 2 ladies (4 marks)
ii) At least 2 ladies (4 marks)
iii) More ladies than men (5 marks)

**Question 4 (20 marks)**

a) State Baye's Theorem (1 mark)

b) State the difference between permutation and combination. (2 marks)

c) A court appoints three jury members from a group of 20 eligible candidates of which 6 are women and 14 are men. If the selection is random, what is the probability that three female jurors are selected? (2 marks)

d) A student answers a multiple-choice examination question that offers four possible answers. Suppose the probability that the student knows the answer to the question is 0.8 and the probability that the student will guess is 0.2. Assume that if the student guesses, the probability of selecting the correct answer is 0.25. If the student correctly answers a question, what is the probability that the student really knew the correct answer? (2 marks)

e) Suppose that A and B are independent events such that the probability that neither occurs is a and the probability of B is b.

Show that P (A) = $\frac{1-b-a}{1-b}$. (3 marks)

f) Y is a discrete random variable having its probability distribution as shown in the table below.

| Y | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| P(y) | k | 2k | 2k | 3k | $k^2$ | $2k^2$ | $7k^2$+ k |

Find i) the value of k (5 marks)

ii) p(y = 6) (2 marks)

iii) p(y ≤ 5) (3 marks)

iv) p(y ≥ 6) (3 marks)

**Question 5(20 marks)**

a) Let Y be a random variable with probability density function

$$f(y) = \begin{cases} k(2-y), & 0 \le y \le 2 \\ 0, & \text{elsewhere} \end{cases}$$

where k is a positive constant.

(i) Find k such that *f(y)* is a valid probability density function. (2 marks)

(ii) Find the cumulative distribution function for Y. Make sure to specify *F(y)* for all real values of y. (4 marks)

(iii) Sketch the graphs for *f(y)* and *F(y)*. (2 marks)

(iv) Find E(Y) and V(Y) (6 marks)

b) Let X be a random variable in mass (in kg) with probability density function (pdf):

$$f(x) = \begin{cases} \frac{1}{36} y(6-y), & 0 \leq y \leq 6 \\ 0, & \text{Otherwise} \end{cases}$$

Findthe probability that the mass is more than 5kg. (5 marks)

c) The eleven letters of the word **MISSIISSIPPI**are all written on a card and the cards shuffled and placed in a line. Compute the number of different arrangements. (3 marks)