KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS
2016/2017 ACADEMIC YEAR

FIRST YEAR FIRST SEMESTER
SPECIAL EXAMINATIONS

FOR THE DEGREE OF MASTER OF SCIENCE IN PHYSICS

COURSE CODE: SPH 810

COURSE TITLE: CLASSICAL MECHANICS

DURATION: 3 HOURS

DATE: 14TH SEPTEMBER 2017 **TIME:** 9:00 AM – 12:00PM

INSTRUCTIONS TO CANDIDATES

- Answer **QUESTION ONE** (Compulsory) and any other two (2) Questions.
- Indicate **answered questions** on the front cover.
- Start every question on a new page and make sure question's number is written on each page.

This paper consists of **3** printed pages. Please Turn Over

QUESTION ONE (30 MARKS)

a) Give a brief account of the history, failures and directions of classical mechanics (3 marks)

b) State the three conservative theorems for a single particle dynamics (3 marks)

c) State and explain the two types of constraints for any given particle system (2 marks)

d) Define elastic and inelastic collisions giving examples of each case. (2 marks)

e) Discuss the principle of conservation of energy and momentum (3 marks)

f) Show that for a single particle with constant mass the equation of motion implies the following differential equation for kinetic energy $\frac{dT}{dt} = F.v$ while if the mass varies with time the corresponding equation is $\frac{d(mT)}{dt} = F.p$ (3 marks)

g) Prove that the magnitude R of the position vector for the center of mass from an arbitrary origin is given by the equation:

$$M^2R^2 = M\sum_i m_i r_i^2 - \frac{1}{2}\sum_{i,j} m_i m_j r_{ij}^2$$ (3 marks)

h) Write D'Alembert's principle. Discuss how dynamics get converted to statics. (3 marks)

i) A particle of mass m moves in one dimension such that it has the Lagrangian

$$L = \frac{m^2\dot{x}^4}{12} - m\dot{x}^2V(x) - V^2(x)$$

where V is some differentiable function of x. Find the equation of motion for$x(t)$ anddescribe the physical nature of the system on the basis of this system. (3 marks)

j) Consider a uniform thin disk that rolls without slipping on a horizontal plane. A horizontal plane is applied to the centre of the disk and in the direction parallel to the plane of the disk.

(i) Derive the Lagrange's equation and find the generalized force. (2 marks)

(ii) Discuss the motion if the force is not applied parallel to the plane of the disk.(3marks)

QUESTION TWO (20 MARKS)

Assume that a particle of mass M_1 moving velocity V_1 collides with a particle of mass M_2 at rest and sticks to it. Describe the motion of mass $M=M_1+M_2$ after collision. Calculate the ratio of K_f(final kinetic energy) and K_i (initial kinetic energy), and discuss that the collision is inelastic. Give other examples of such a collision. (20 marks)

QUESTION THREE (20MARKS)

a) Obtain the equation of motion for a particle falling vertically under the influence of gravity when the frictional forces obtainable from a dissipation function $\frac{1}{2}kv^2$are present. Integrate the equation to obtain the velocity as a function of time and show that the maximum possible velocity for a fall from rest is $v + \frac{mg}{k}$. (14 marks)

b) Describe the motion of a particle in a plane in polar coordinates. (6 marks)

QUESTION FOUR (20 MARKS)

(a) Solve the problem of the motion of a point projectile in a vertical plane, usingthe Hamilton-Jacobi method. Find both the equation of the trajectory and thedependence of the coordinates on time, assuming the projectile is fired off at time $t = 0$ from the origin with the velocity v_o, making an angle θ with the horizontal. (10 marks)

(b) Show by use of Poisson brackets that for one-dimensional harmonic oscillator, there is a constant of the motion μ defined as $\mu(q,p,t)=\ln(p+im\omega q)-i\omega t$, $\omega = \sqrt{\frac{k}{m}}$

What is the physical significance of this constant of motion? (10 marks)

QUESTION FIVE (20 MARKS)

The transformation equations between two sets of equations are

$Q=\log(1 + q^{1/2}\cos(p))$

$P=2(1 + q^{1/2}\cos(p))\, q^{1/2} \sin(p)$

a) Show directly from these transformation equations that Q, P are canonical variables if q and p are. (10 marks)

k) Show that the function that generates this transformation is $F_3=-(e^Q-1)^2\tan(p)$. (10marks)