KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS
2017/2018 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER
SPECIAL / SUPPLEMENTARY EXAMINATIONS

FOR THE DEGREE OF BACHELOR OF SCIENCE IN PHYSICS

COURSE CODE: **SPH 414**

COURSE TITLE: **QUANTUM MECHANICS II**

DURATION: 2 HOURS

DATE: 03/10/2018 **TIME: 8.00 – 10.00 AM**

INSTRUCTIONS TO CANDIDATES

- Answer **QUESTION ONE** (Compulsory) and any other **TWO** (2) Questions.
- Question **ONE** carries **30 MARKS** and the remaining carry **20 MARKS** each.
- Symbols used bear usual meaning.

Instructions

The following rules of commutator algebra may be used where necessary

$$[\hat{A},\hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A} = -[\hat{B},\hat{A}]$$
$$[\hat{A},\hat{B}+\hat{C}] = [\hat{A},\hat{B}] + [\hat{A},\hat{C}]$$
$$[\hat{A}+\hat{B},\hat{C}] = [\hat{A},\hat{C}] + [\hat{B},\hat{C}]$$
$$[\hat{A},\hat{B}\hat{C}] = [\hat{A},\hat{B}]\hat{C} + \hat{B}[\hat{A},\hat{C}]$$
$$[\hat{A}\hat{B},\hat{C}] = [\hat{A},\hat{C}]\hat{B} + \hat{A}[\hat{B},\hat{C}]$$
$$\left[\hat{A},[\hat{B},\hat{C}]\right] + \left[\hat{B},[\hat{C},\hat{A}]\right] + \left[\hat{C},[\hat{A},\hat{B}]\right] = 0$$

QUESTION 1 (30 MARKS)

a) Show that $[x, p] = i\hbar$ (4 marks)

b) Show that orbital angular momentum is represented in the position representation of wave mechanics by the vector operator $\vec{L} = -i\hbar\,(\vec{r} \times \nabla)$ (5 marks)

c) By considering L_x, L_y, and L_z, show that angular momentum L is self-adjoint (4 marks)

d) Define the term perturbation? (1 mark)

e) List two categories of perturbation theory? (2 marks)

f) Reduce the following arbitrary product of spin ½ operators?

i) $S_xS_yS_zS_yS_zS_x$ (4 marks)

ii) $S_xS_yS_xS_yS_zS_x$ (4 marks)

g) Show that

i. $s^2\alpha = \frac{3}{4}\hbar^2\alpha$ (3 marks)

ii. $s_z\alpha = \frac{1}{2}\hbar\alpha$ (3 marks)

QUESTION TWO (20 MARKS)

Show that

(a) $[L_x, y] = i\hbar z$ (10 marks)

(b) $[L_x, p_y] = i\hbar p_z$ (10 marks)

QUESTION THREE (20 MARKS)

Show that

a) $[L_x, L_y] = i\hbar L_z$ (10 marks)

b) $[L^2, L_x] = 0$ (10 marks)

QUESTION FOUR (20 MARKS)

Show that

a) $J^2 = \frac{3}{4}\hbar^2\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ (10 marks)

b) $J_z = \frac{1}{2}\hbar\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$ (10 marks)

QUESTION FIVE (20 MARKS)

The matrices representing S_x, S_y, and S_z, which acts on the spin wave function; $S = \frac{1}{2}$, are $s = \frac{1}{2}\hbar\sigma$ with $\sigma_x = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$, $\sigma_y = \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}$ and $\sigma_z = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$. Show that $[\sigma_x, \sigma_y] = 2i\sigma_z$, $[\sigma_z, \sigma_x] = 2i\sigma_y$ and $[\sigma_y, \sigma_z] = 2i\sigma_x$ (20 marks)