KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS
2017/2018 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER
MAIN EXAMINATIONS

FOR THE DEGREE OF BACHELOR OF SCIENCE IN PHYSICS

COURSE CODE: SPH 414

COURSE TITLE: QUANTUM MECHANICS II

DURATION: 2 HOURS

DATE: 3/8/2018 **TIME:** 2-4PM

INSTRUCTIONS TO CANDIDATES

- Answer **QUESTION ONE** (Compulsory) and any other **TWO** (2) Questions.
- Question one carries 30 MARKS while all the other questions 20 MARKS each.

The following rules of commutators algebra may be used

- $[A, B] = AB - BA = -[B, A]$
- $[A + B, C] = [A, C] + [B, C]$
- $[AB, C] = A[B, C] + [A, C]B$

QUESTION ONE(30 MARKS)

a) Mathematically define the quantized linear momentum operator **(1 mark)**

b) By considering the mathematical definition of angular momentum and the quantized linear momentum operator, expressL_x, $L_y$and L_z in terms of $\hbar$? **(9 marks)**

c) Solve the following commutation $[p_x, x]$ **(5 marks)**

d) Given that the spin for electrons is given by the matrices

$$s_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, s_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \text{ and } s_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

By noting that $i^2 = -1$, show that

$i)\ [s_x, s_y] = i\hbar s_z$ **(5 marks)**

$ii)\ [s_y, s_z] = i\hbar s_x$ **(5 marks)**

$iii)\ [s_z, s_x] = i\hbar s_y$ **(5 marks)**

QUESTION TWO (20 MARKS)

Find the commutators:

(i). $[X, L_y]$ **(3 marks)**

(ii). $[X, L_z]$ **(3 marks)**

(iii). $[P_x, L_y]$ **(4 marks)**

(iv). $[P_x, L_z]$ **(4 marks)**

(v). $[ZP_x, L_z]$ **(6 marks)**

QUESTION THREE (20 MARKS)

Given the fact that$[\vec{J}_l, \vec{J}_m] = i\hbar\vec{J}_n$ where $l, m, n = 1, 2, 3$ are the permutation relations in the first, second and third dimension. Show that

a) $[\vec{J}_x^2, \vec{J}_y] = -\hbar^2\vec{J}_y + 2i\hbar\vec{J}_x\vec{J}_z$ **(5 marks)**

b) $[\vec{J}_z^2, \vec{J}_y] = \hbar^2\vec{J}_y - 2i\hbar\vec{J}_x\vec{J}_z$ **(5 marks)**

c) $[J^2, \vec{J}_y] = 0$ **(5 marks)**

d) $[J^2, \vec{J}_x] = 0$ **(5 marks)**

QUESTION FOUR (20 MARKS)

Given that$\hat{L}_\pm$ and$\hat{R}_\pm$ are defined by $\hat{L}_\pm = \hat{L}_x \pm i\hat{L}_y$ and$\hat{R}_\pm = \hat{X} \pm i\hat{Y}$. Prove the following

a) $[\hat{L}_+, \hat{R}_+] = 0$ **(10 marks)**

b) $[\hat{L}_+, \hat{R}_-] = 2\hbar\hat{Z}$ **(10 marks)**

QUESTION FIVE (20 MARKS)

One description of spin 1 particles uses the matrices $M_x = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$,

$M_y = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}$, and $M_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$ Show that

a) $[M_x, M_y] = iM_z$ **(7 marks)**

b) $M^2 \equiv M_x^2 + M_y^2 + M_z^2 = 2I$ Where I is a unit matrix **(7 marks)**

c) $[M^2, M_y] = 0$ **(6 marks)**