KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS
2017/2018 ACADEMIC YEAR

THIRD YEAR SECOND SEMESTER
SUPPLEMENTARY / SPECIAL EXAMINATIONS

FOR THE DEGREE OF BACHELOR OF SCIENCE IN PHYSICS AND BACHELOR OF EDUCATION (SCIENCE)

COURSE CODE: SPH 313

COURSE TITLE: QUANTUM MECHANICS I

DURATION: 2 HOURS

DATE: 10/10/ 2018. **TIME:** 11:30-1:30PM

- Answer question **ONE** and any other **TWO** of the remaining
- Question **ONE** carries **30 MARKS** and the remaining carry **20 MARKS** each.
- Symbols used bear usual meaning.
- Plancks constant $h = 6.63 \times 10^{-34} JS$
- $1eV = 1.6 \times 10^{-19} J$
- Speed of light, $c = 3.0 \times 10^{8} ms^{-1}$
- Rest mass, $m_e = 9.91 \times 10^{-31} kg$
- Mass of proton, $m_p = 1.67 \times 10^{-27} Kg$

QUESTION ONE (30 MARKS)

(a) What is the energy of a photon of visible light of wavelength $\lambda = 6 \times 10^{-7} m$ **(4 mks)**

(b) When two ultra violet beams of wavelengths $\lambda_1 = 280nm$ and $\lambda_2 = 490nm$ fall on a lead surface they produce photoelectrons with maximum energies 8.57eV and 6.67eV, respectively

i). Estimate the numerical value of the Planck's constant **(3 mks)**

ii). Calculate the work function of lead **(3 mks)**

iii). Calculate the cut off frequency of lead **(3 mks)**

(c) From the photoelectric experiment explain how the classical theory failed to account for experimental results **(6 mks)**

(d) Calculate the De Broglie's wavelength for a mass of 2g moving at a speed of 1m/s **(3 mks)**

(e) Given a wave function $\Psi(x,t) = A\, e^{i(kx-\omega t+\theta)}$, the classical definition of total energy and by considering the De Broglie's relations, derive time dependent Schrodinger equation along the x-axis? **(8 mks)**

QUESTION TWO (20 MARKS)

a) Give the difference between commutative quantum operators and non-commutative quantum operators **(4 mks)**

b) Prove that for the operators A, B, and C, the following identities are valid:

(i). $[\hat{B},\hat{A}] = -[\hat{A},\hat{B}]$ **(3 mks)**

(ii). $[\hat{A}+\hat{B},\hat{C}] = [\hat{A},\hat{C}] + [\hat{B},\hat{C}]$ **(3 mks)**

c) Derive the energy of an electron in the hydrogen atom using Bohr's formulas **(7 mks)**

d) Calculate De Broglie's wavelength for a proton of kinetic energy 70MeV. **(3mks)5c**

QUESTION THREE (20 MARKS)

a) Consider fig.1 showing the insight of Compton Effect. Find the increase in the photon's wavelength as a function of the scattering angle θ. **(10 mks)**

Fig. 1

b) The wavelength and the frequency in a wave guide are related by $\upsilon = \sqrt{\frac{2\pi T}{\rho\lambda^3}}$ (water waves in shallow water; T is the surface tension and ρ the density). Express the group velocity v_g in terms of the phase velocity $\boldsymbol{v_p} = \lambda\vartheta$ **(10 mks)**

QUESTION FOUR (20 MARKS)

a) Consider a particle trapped in a well with potential given by:

$$V(x) = \begin{cases} 0 & 0 \le x \le a \\ \infty & otherwise \end{cases}$$

Show that $\Psi(x,t) = A\,sin\,(kx)\exp - \left(\frac{iEt}{\hbar}\right)$ solves the Schrödinger equation provided that $E = \frac{\hbar^2 k^2}{2m}$ **(10mks)**

b) Suppose $\Psi(x,t) = A(x - x^3)e^{-iEt/\hbar}$. Find $V(x)$ such that the Schrödinger equation is satisfied. **(10 mks)**

QUESTION FIVE (20 MARKS)

Consider a particle subjected to a time-independent potential $V(r)$.

a) Assume that a state of the particle is described by a wave function of the form $\Psi(r,t) = \Phi(r)\chi(t)$. Show that $\chi(t) = Ae^{-i\omega t}$ (A is constant) and that $\Phi(r)$ must satisfy the equation

$$-\frac{\hbar}{2m}\nabla^2\Phi(r) + V(r)\Phi(r) = \hbar\omega\Phi(r)$$

Where m is the mass of the particle **(10 mks)**

b) Show that for a one-dimensional square - integrable wave-packet,

$$\int_{-\infty}^{\infty} j(x)dx = \frac{\langle p \rangle}{m}$$

Where $j(x)$, is the probability current **(10 mks)**6b