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KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS
2017/2018 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER
MAIN EXAMINATIONS

FOR THE DEGREE OF BSC (PHYSICS) AND B.ED (SCIENCE)

COURSE CODE: SPH 312

COURSE TITLE: CLASSICAL MECHANICS II

DURATION: 2 HOURS

DATE: 10TH JANUARY 2018 TIME: 9 – 11AM

INSTRUCTIONS TO CANDIDATES

- Answer **QUESTION ONE** (Compulsory) and any other two (2) Questions.
- Indicate **answered questions** on the front cover.
- Start every question on a new page and make sure question's number is written on each page.
- Symbols have their usual meaning.

This paper consists of **4** printed pages. Please Turn Over

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QUESTION ONE (30 MARKS)

(a). Two blocks A and B of mass M_A and M_B resting of a frictionless surface are connected by a light spring. The masses are pulled apart so that the spring is stretched after which they are released. Show that;

(a). $\vec{V}_A = \frac{M_A}{M_B}\vec{V}_B$ where $\vec{V}$ is velocity (2mark)

(b). $\frac{K_A}{K_B} = \frac{M_B}{M_A}$ where $K$is kinetic energy(2marks)

(b) Given the following Lagrangian, compute the equation of motion it represents

$L = 0.5ml^2\dot{\theta}^2 + mlr\omega^2\sin(\theta - \omega t) + mglcos\theta$where l and r are constants. (5marks)

(c) Consider a particle of mass m moving in the xy-plane such that its position vector is $\boldsymbol{r} = acos\omega t\boldsymbol{i} + bsin\omega t\boldsymbol{j}$ where a, b and ω are positive constants and $a > b$

(i) Prove that the particle moves in an elliptical path. **(2marks)**

(ii) Show that the force acting on the particle is always directed towards the origin. **(2marks)**

(d). If $\vec{R}_{CM}$ is the position vector of the center of mass of a system of particles with respect to the origin O, $\vec{r}_i$ is the position vector of the i-th particle and $\vec{P}_i$ is the momentum of the i-th particle, show that the angular momentum of the system is;

$\vec{L} = \left[\vec{R}_{CM} \times MV_{CM} + \sum_i \vec{r}_i^{\,'} \times \vec{P}_i^{\,'}\right]$ **(5marks)**

(e) Find the velocity and position of the center of mass when the only external forces are due to the uniform gravitational field. **(2marks)**

(f) Show that the moment of inertia of an annular cylinder is given as $I = \frac{1}{2}M(R_1^2 + R_2^2)$where$M$ is the total mass,R_1 is the inner radius and R_2 is the outer radius.

(4marks)

(g) A body experiences zero gravity when it is placed at a distance r_e from the earth along the line joining the earth and the sun. The sun is at a distance if $1.5 \times 10^5 km$ from the earth. The mass of the sun is $3.24 \times 10^5 M_e$ where M_e represents the mass of the earth. Determine the value r_e. **(4marks)**

(h) Distinguish between holomonic and non-holomonic constraints. **(2marks)**

QUESTION TWO (20 MARKS)

a) An alpha particle (Helium atom) is emitted from Uranium-238 nucleus originally at rest with a speed of $1.4 \times 10^7 m/s$ and a kinetic energy of 4.1 Mev. Find the recoil the recoil speed of the residual nucleus which is Thorium-234 and the kinetic energy of the Thorium. (6 marks)

b) Considering a particle of mass m at a distance r' from the center of the earth and moving towards the center of the earth, show that the gravitational potential $V(r)$ is given by;

$$\boldsymbol{V(r) = -\frac{GM}{r}}$$(5marks)

c) Using the result of (b) above, show that the gravitational potential at a point P on a sphere of radius a is given by; $V = \frac{2\pi a G\sigma}{r}[a + r \mp (r - a)]$ where σ is the surface density of the sphere.(4marks)

d) Use the Lagrangian formulation to obtain the equation of motion of a particle falling vertically under the influence of gravity when frictional forces obtainable from a dissipative function $\frac{1}{2}kv^2$ are present.Hence show that the maximum possible velocity for fall from rest is $v = \frac{mg}{k}$. (5marks)

QUESTION THREE (20MARKS)

(a) State the important properties of central force fields. Give an example of a central force field. **(2marks)**

(b) Discuss one special case where the force applied on a body causes it to move but no work is done. **(3marks)**

(c) Consider an object of mass m located at a distance h above the earth's surface or a distance r from the earth's center, where $r = R_E + h$. Derive an equation to show that the acceleration g due to gravity decreases with increasing altitude. **(4marks)**

(d) State Kepler's three laws of planetary motion **(3marks)**

(e) A rocket of initial total mass $4.0 \times 10^4 Kg$ ejects hot gases from the combustion chamber at a rate of 1000Kg/s. The speed of the gas molecules ejected is 2.0 Km/s relative to the rocket. The fuel which makes 90% of the total mass of the rocket before blast off is consumed in 36s.

i. Find the thrust of the rocket engine (2marks)
ii. Find the upward acceleration at time t=0, t=20, t=36s. (6marks)

QUESTION FOUR (20 MARKS)

(a). A pendulum bob of mass $m$attached on a thread of length l makes an angle θ with the horizontal. When released, the bob is constrained to move in a vertical plane with only the angle coordinate describing the motion. Using the Lagrangian formulation,

i. Show that the equation of motion for the system is $\ddot{\theta} + \frac{g}{l}\theta = 0$ (7marks)

ii. Show that the periodic time $T = 2\pi\sqrt{\frac{l}{g}}$ (3marks)

(b) Considering a conservative, holonomic, dynamical system that moves from a point P to another point Q at times $t_1$and t_2 respectively, prove the Hamilton's principle, i.e $\int_1^2 Ldt = J = extremium.$ **(10marks)**

QUESTION FIVE (20 MARKS)

(a). Let $\vec{r}_v$ and $\vec{v}_v$ be the position vector and velocity, respectively, of the v_th particle relative to the centre of mass. Prove that;

(a). $\sum_v m_v \vec{r}_v = 0$ (3marks)

(b). $\sum_v m_v \vec{v}_v = 0$(3marks)

(b) Define moment of inertia of a rotating body. **(1mark)**

(c) Calculate the moment of inertia of a uniform rigid rod of length L and mass M about an axis perpendicular to the rod and passing through its center of mass. Also find the moment of inertia about the y'axis. **(6marks)**

(d) Estimate the magnitude of the angular momentum of a solid ball of mass 6kg that is spinning at 10rev/s. **(3marks)**

(e) Write down the parallel and perpendicular axis theorems. **(2marks)**

(f) Consider a circular disc of mass M and radius R rolling down an inclined plane without slipping. Find the speed of its center of mass when it reaches the bottom of the incline. **(3marks)**