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KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS
2016/2017 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER
SUPPLEMENTARY/SPECIAL EXAMINATIONS

FOR THE DEGREE OF BSC (PHYSICS) AND B.ED (SCIENCE)

COURSE CODE: **SPH 310**

COURSE TITLE: **MATHEMATICAL PHYSICS I**

DURATION: 2 HOURS

11th 8—10 AM

DATE: MONDAY ~~9TH~~ SEPTEMBER 2017 TIME: ~~9 – 11~~ AM

INSTRUCTIONS TO CANDIDATES

- Answer **QUESTION ONE** (Compulsory) and any other two (2) Questions.
- Indicate **answered questions** on the front cover.
- Start every question on a new page and make sure question's number is written on each page.
- Symbols have their usual meaning.

This paper consists of **3** printed pages. Please Turn Over

KIBU observes ZERO tolerance to examination cheating

QUESTION ONE 30marks

(a) Given that $\boldsymbol{A} = 6\boldsymbol{i} + 4\boldsymbol{j} + 3\boldsymbol{k}$ and $\boldsymbol{B} = 2\boldsymbol{i} - 3\boldsymbol{j} - 3\boldsymbol{k}$, show that $\boldsymbol{A}.\boldsymbol{A} \times \boldsymbol{B} = 0$ (3marks)

(b) For a particle moving in a circular orbit with its radius given as $\boldsymbol{r} = \boldsymbol{i}r\cos wt + \boldsymbol{j}r\sin w$ show that $\ddot{\mathbf{r}} + \omega^2 \boldsymbol{r} = 0$ (4marks)

(c) Find if the matrix element A defined as $A = \begin{pmatrix} 1 & 5 & 7 \\ 5 & 3 & -4 \\ 7 & -4 & 0 \end{pmatrix}$ is symmetric(3marks)

(d) Show that the matrix $H = \begin{pmatrix} 1 & -i \\ i & 1 \end{pmatrix}$ is a Hermitian (3marks)

(e) Given that $\boldsymbol{u} = x^2\boldsymbol{i} + y^3\boldsymbol{j} + z\boldsymbol{k}$, find

- **i.** $\boldsymbol{\nabla u}$ (2marks)
- **ii.** $\boldsymbol{\nabla . u}$ (2marks)
- **iii.** $\boldsymbol{\nabla \times u}$ (3marks)
- iv. $\boldsymbol{\nabla^2 u}$ (3marks)

(f) The vertices of a triangle AB and Care given by the points (-1,0,2),(0,1,0) and (1,-1,0), respectively. Find point D such that the figure forms a plane parallelogram (3marks)

(g) What is the size of the angle between vectors $\boldsymbol{A} = 3\boldsymbol{i} + 4\boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{B} = \boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k}$ (2marks)

(h) Calculate $\nabla.\vec{r}$for a position vector (2marks)

QUESTION TWO 20marks

i. If$S(x, y, z) = (x^2 + y^2 + z^2)^{-3/2}$, find

(a) $\boldsymbol{\nabla}$**S**at the point (1, 2, 3) (6marks)

(b) The magnitude of the gradient of S, $|\boldsymbol{\nabla}\mathbf{S}|$ at (1, 2, 3)(4marks)

ii. Show that, $\nabla.\nabla\times V = 0$, if V = ($V_x$i+ V_yj+ V_zk) (5marks)

iii. Show that the gradient of any scalar field $\emptyset(r)$ is irrotational(5marks)

QUESTION THREE 20marks

a) Given that $x = \rho \cos \varphi$ and $y = \rho \sin \varphi$ for polar coordinate system and by using the jacobian find the area element of a polar coordinate system (8marks)

b) Given that $x = r \sin \theta \cos \varphi, y = r \sin \theta \sin \varphi$ and $z = \cos \theta$ for spherical coordinate system. By using jacobian find the area element of a spherical coordinate system(10marks)

c) State stokes theorem (2marks)

QUESTION FOUR 20marks

a) Given a vector $\boldsymbol{r}$in the x-y axis. If $\boldsymbol{r}$ has a fixed direction and the Cartesian coordinate is rotated in the counter-clockwise direction about $z -$ axis through angle q, such that we have x'-y' coordinate axes. Show that:

$x' = x\cos\theta + y\sin\theta$

$y' = -x\sin\theta + y\cos\theta$

Hence show that **r** is invariant under rotation(12marks)

b) Find the Laplace Transforms of each of the following.

i. $f(x) = 3e^{-4x}$

ii. $f(x) = 4 \cos 5x$ (8marks)

QUESTION FIVE 20marks

a) A force is described by $\mathbf{F} = -\boldsymbol{i}\frac{y}{x^2+y^2} + \boldsymbol{j}\frac{x}{x^2+y^2}$calculate the divergence of F,and the curl of F (10marks)

b) Find Fourier Transform of.$f(x) = \begin{cases} 1 & |x| < a \\ 0 & |x| > a \end{cases}$ (6marks)

c) Prove Gauss' theorem (4marks)