KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS
2017/2018 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER
SPECIAL / SUPPLEMENTARY EXAMINATIONS

FOR THE DEGREE OF BSC (PHYSICS) AND B.ED (SCIENCE)

COURSE CODE: **SPH 310**

COURSE TITLE: **MATHEMATICAL PHYSICS I**

DURATION: 2 HOURS

DATE: 01/10/2018 **TIME: 8-10 AM**

INSTRUCTIONS TO CANDIDATES

- Answer **QUESTION ONE** (Compulsory) and any other two (2) Questions.
- Indicate **answered questions** on the front cover.
- Start every question on a new page and make sure question's number is written on each page.
- Symbols have their usual meaning.

This paper consists of **3** printed pages. Please Turn Over

KIBU observes ZERO tolerance to examination cheating

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QUESTION ONE (30 MARKS)

(a) If $\boldsymbol{A} = i + 2j - 3k$ and $\boldsymbol{B} = 3i - j + 2k$ find

1) $\mathbf{3(3A - 2B) + 2(3B - 2A)}$ **(2 marks)**
2) $\mathbf{(4A).(2B)}$ **(3 marks)**
3) $\mathbf{3A \times 2B}$ **(3 marks)**

(b) If $\vec{a}$ and $\vec{b}$ be two unit vectors and α be the angle between them, then find the value of α such that $\vec{a} + \vec{b}$ is a unit vector **(4 marks)**

(c) Given that $\boldsymbol{u} = x^2\boldsymbol{i} + y^2\boldsymbol{j} + z\boldsymbol{k}$, find

i. $\boldsymbol{\nabla . u}$ **(2 marks)**

ii. $\boldsymbol{\nabla \times u}$ **(3 marks)**

(d) Given that $\boldsymbol{A} = 3\boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{B} = 2\boldsymbol{i} - \boldsymbol{j} - \boldsymbol{k}$, show that $\boldsymbol{A}.\boldsymbol{A} \times \boldsymbol{B} = 0$ **(3 marks)**

(e) Prove that: $\hat{\imath} \times (\vec{A} \times \hat{\imath}) + \hat{\jmath} \times (\vec{A} \times \hat{\jmath}) + \hat{k} \times (\vec{A} \times \hat{k}) = 2\vec{A}$, where $\vec{A} = A_1\hat{\imath} + A_2\hat{\jmath} + A_3\hat{k}$ **(5 marks)**

(f) Determine λ such that $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$, $\vec{b} = 2\hat{\imath} - 4\hat{k}$, and $\vec{c} = \hat{\imath} + \lambda\hat{\jmath} + 3\hat{k}$ are coplanar **(5 marks)**

QUESTION TWO (20 MARKS)

(a) A vector **r** is in the x-y Cartesian coordinate. If **r** has a fixed direction and the Cartesian coordinate is rotated in the counter-clockwise direction about z axis through angle q, such that we have $x' - y'$ coordinate axes. By using a diagram show that:

$$x' = x\cos\theta + y\sin\theta$$
$$y' = -x\sin\theta + y\cos\theta$$ **(4marks)**

Hence show that **r** is invariant under rotation **(3marks)**

(b) State Gauss' theorem **(2marks)**

(c) Prove Gauss' theorem stated in 2(b) above **(5marks)**

(d) One description of spin 1 particles uses the matrices $M_x = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$, $M_y = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}$, and $M_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$ Show that $[M_x, M_y] = iM_z$ **(6 marks)**

QUESTION THREE (20 MARKS)

(i) If $S(x, y, z) = (x^2 + y^2 + z^2)^{-3/2}$, find

(a) $\boldsymbol{\nabla}\mathbf{S}$ at the point (1, 2, 3) **(4 marks)**

(b) The magnitude of the gradient of S, $|\boldsymbol{\nabla}\mathbf{S}|$ at (1, 2, 3) **(2 marks)**

(ii) Show that, $\boldsymbol{\nabla}.\boldsymbol{\nabla} \times \mathbf{V} = 0$, if $V = V_x\hat{\imath} + V_y\hat{\jmath} + V_z\hat{k}$ **(4 marks)**

(iii) Show that the gradient of any scalar field $\phi(r)$ is Irrotational **(4 marks)**

(iv) One description of spin 1 particles uses the matrices $M_x = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$, $M_y = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}$, and $M_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$ Show that $M^2 \equiv M_x^2 + M_y^2 + M_z^2 = 2I$

Where I is a unit matrix **(6 marks)**

QUESTION FOUR (20 MARKS)

(i) Given that $x = \rho\cos\varphi$ and $y = \rho\sin\varphi$ for polar coordinate system. By using the Jacobian, find the area element of a polar coordinate system **(7marks)**

(ii) Given that $x = r\sin\theta\cos\varphi$, $y = r\sin\theta\sin\varphi$, and $z = r\cos\theta$ for spherical coordinate system. By using the Jacobian, find the area element of a spherical coordinate system **(7marks)**

(iii) One description of spin 1 particles uses the matrices $M_x = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$, $M_y = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}$, and $M_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$ Show that $[M^2, M_y] = 0$ **(6marks)**

QUESTION FIVE (20 MARKS)

(i) A force is described by

$$\mathbf{F} = -i\frac{y}{x^2+y^2} + j\frac{x}{x^2+y^2}$$

(a) Calculate the divergence of **F** **(4 marks)**

(b) calculate the curl of **F** **(6 marks)**

(ii) A particle moving in a circular orbit is given by a vector $\boldsymbol{r} = \boldsymbol{i}r\cos\omega t + \boldsymbol{j}r\sin\omega t$. Evaluate $\boldsymbol{r} \times \dot{\boldsymbol{r}}$, where r is the radius and ω is the angular velocity and both are constants **(4 marks)**

(iii)Prove that: $\vec{A} \times (\vec{B} \times \vec{C}) + \vec{B} \times (\vec{C} \times \vec{A}) + \vec{C} \times (\vec{A} \times \vec{B}) = 0$ **(6 marks)**