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KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS
2017/2018 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER
MAIN EXAMINATIONS

FOR THE DEGREE OF BSC (PHYSICS) AND B.ED (SCIENCE)

COURSE CODE: SPH 310

COURSE TITLE: MATHEMATICAL PHYSICS I

DURATION: 2 HOURS

DATE: 8^{TH} JANUARY 2018 **TIME:** 9 – 11AM

INSTRUCTIONS TO CANDIDATES

- Answer **QUESTION ONE** (Compulsory) and any other two (2) Questions.
- Indicate **answered questions** on the front cover.
- Start every question on a new page and make sure question's number is written on each page.
- Symbols have their usual meaning.

This paper consists of **4** printed pages. Please Turn Over

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QUESTION ONE (30 MARKS)

a) If $\vec{A} = 10\hat{\imath} - 4\hat{\jmath} + 6\hat{k}$ and $\vec{B} = 2\hat{\imath} + \hat{\jmath}$, find:

(i) the component of $\vec{A}$ along y - axis **(1 mark)**

(ii) themagnitude of $3\vec{A} - \vec{B}$ **(3 marks)**

(iii) a unit vector along $\vec{A} + 2\vec{B}$ **(3 marks)**

(iv) show that the magnitude of unit vector in a) (iii) above is one **(2 marks)**

b) Given that $\vec{a}$ and $\vec{b}$ can be expressed as unit vectors as shown

$$\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$$

$$\vec{b} = b_x\hat{\imath} + b_y\hat{\jmath} + b_z\hat{k}$$

Show that $\vec{a}.\vec{b} = (a_x b_x) + (a_y b_y) + (a_z b_z)$ **(4 marks)**

c) What is the angle between $\hat{\imath} - 2\hat{\jmath}$ and $\hat{\imath} + \hat{\jmath}$ **(3 marks)**

d) Point P and Q are located at (0, 2, 4) and (-3, 1, 5) respectively, calculate

i. The position vector P **(1 mark)**

ii. The distance vector from P to Q **(3 marks)**

iii. The distance between P and Q **(2 marks)**

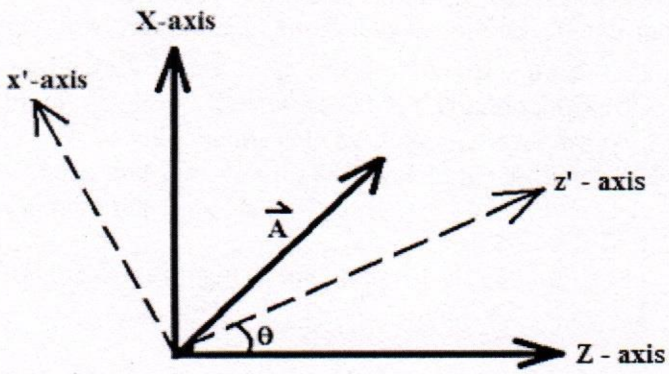
iv. A vector parallel to PQ with magnitude 10. **(4 marks)**

e) Find if the matrix A defined as $A = \begin{pmatrix} 1 & 5 & 7 \\ 5 & 3 & -4 \\ 7 & -4 & 0 \end{pmatrix}$ is symmetric **(2marks)**

f) Show that the matrix $\mathbf{H} = \begin{pmatrix} 1 & -i \\ i & 1 \end{pmatrix}$ is an example of a Hermitian matrix **(2marks)**

QUESTION TWO (20 MARKS)

a) The following figure shows vector $\vec{A}$ in a two dimension Cartesian plane. If the plane is rotated through angle θ counterclockwise, express A'_y and A'_z in terms of A_y, A_z and θ **(5 marks)**



b) Show that A_y and A_z show form invariance **(5 marks)**

c) Show that $\vec{B}.\vec{A} \times \vec{B} = 0$ **(4 marks)**

d) Define orthogonal matrix **(2 marks)**

e) Find if the matrix $A = \begin{pmatrix} 3 & -1 \\ -2 & 1 \end{pmatrix}$ is orthogonal **(2 marks)**

QUESTION THREE (20 MARKS)

(a) Show whether the set of vectors defined as $\vec{A} = 2\hat{\imath} + 2\hat{\jmath} - 2\hat{k}$, $\vec{B} = \hat{\imath} - \hat{\jmath} + \hat{k}$ and $\vec{C} = 3\hat{\imath} + \hat{\jmath} + \hat{k}$ are coplanar **(4 marks)**

(b) Given that $\boldsymbol{u} = x^2\boldsymbol{i} + y^2\boldsymbol{j} + z\boldsymbol{k}$, find

i. $\nabla . \boldsymbol{u}$ **(2marks)**

ii. $\nabla \times \boldsymbol{u}$ **(2 marks)**

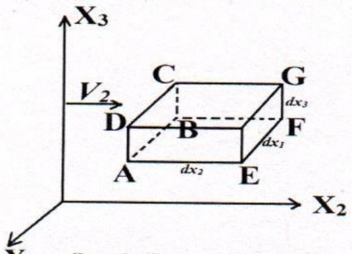
(c) Given the matrices $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 3 & 3 \\ 4 & 1 & 1 \\ 1 & 1 & 2 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 5 \\ 1 & 3 & 1 \end{pmatrix}$ find the Trace of (ABC). **(6 marks)**

(d) By considering matrices A and C in 3(c) above, find $(AC)^{-1}$ **(6 marks)**

QUESTION FOUR (20 MARKS)

a) Differentiate between ∇f and $\nabla . f$ if f is a scalar **(2 marks)**

b) Consider a steady motion of a fluid of density $\rho(x_1, x_2, x_3)$ and the velocity field given by $\vec{V}(x_1, x_2, x_3)$ where $\vec{V}(x_1, x_2, x_3) = V_1(x_1, x_2, x_3)\hat{e}_1 + V_2(x_1, x_2, x_3)\hat{e}_2 + V_3(x_1, x_2, x_3)\hat{e}_3$ passing through a small parallelepiped ABCDEFGH of dimensions dx_1, dx_2, dx_3 as shown below



Steady flow of a fluid

Show that the mass loss per unit time per unit volume is $\nabla . (\rho\vec{V})$ **(6 marks)**

c) In circular cylindrical coordinates the orbital angular momentum takes the form for the coordinate vector $\boldsymbol{r} = \boldsymbol{\rho} + \boldsymbol{z}$ and $\vec{\boldsymbol{V}} = \dot{\rho}\widehat{\boldsymbol{\rho}} + \rho\dot{\varphi}\widehat{\boldsymbol{\varphi}} + \dot{z}\hat{\boldsymbol{z}}$. Given that $\boldsymbol{p} = \boldsymbol{m}\vec{\boldsymbol{V}}$, find the Orbital Angular Momentum '$\boldsymbol{L}$' in Cylindrical Coordinates **(3 marks)**

d) Now let us take the mass to be 3 kg, the lever arm as 1 m in the radial direction of the *xy*-plane, and the velocity as 2 m/s in the *z*-direction. Estimate the value of **L** in terms of $\widehat{\boldsymbol{\varphi}}$ direction **(4 marks)**

e) Find the determinant of the matrix $A = \begin{bmatrix} 1 & 3 & 3 \\ 3 & 2 & -1 \\ -2 & 2 & 1 \end{bmatrix}$ **(5 marks)**

QUESTION FIVE (20 MARKS)

a) (i) If $x = \rho \cos \varphi$ and $y = \rho \sin \varphi$ in polar coordinates form; write the Jacobian for this **(2 marks)**

(ii) Express the Cartesian coordinate $dxdy$ in polar form **(2 marks)**

b) If $x = r \sin \theta \cos \varphi$, $y = r \sin \theta \sin \varphi$ and $z = r\, cos\theta$ in spherical coordinates form;

(i) Write the Jacobian for this **(3 marks)**

(ii) Express the Cartesian coordinate $dxdydz$ in spherical form **(7 marks)**

c) Show that the matrix $\begin{pmatrix} 4 & 5+2i & 6+3i \\ 5-2i & 5 & -1-2i \\ 6-3i & -1+2i & 6 \end{pmatrix}$ is hermitian **(6 marks)**