(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

FIRST YEAR FIRST SEMESTER

SPECIAL/SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF MASTER OF SCIENCE IN APPLIED MATHEMATICS

COURSE CODE: **MAT 858**

COURSE TITLE: NUMERICAL COMPUTATION FOR DIFFERENTIAL EQUATIONS

DATE: 02/10/18 **TIME:** 9 AM -12 PM

INSTRUCTIONS TO CANDIDATES

Answer Any THREE Questions

TIME: 3 Hours

Question 1[20marks]

(a) Construct Crank-Nicolson implicit finite-difference scheme as applied to the second order linear parabolic partial differential equation

$$\frac{\partial u}{\partial t} = 2\psi \frac{\partial^2 u}{\partial x^2}, \qquad a \le x \le b, \qquad t > 0 \quad \psi > 0$$

subject to $u(0,t) = \alpha$, $u(1,t) = \phi \quad t > 0$

and $u(x,0) = \sin 4\pi x \quad a \le x \le b$. [10 marks]

(b) Determine the stability condition for the Crank-Nicolson implicit finite-difference scheme in part (a) above [10marks]

Question 2 [20 marks]

(a) Use a discriminant Δ theory to categorize ; elliptic, parabolic and hyperbolic partial differential equations given below.

(i) $x^2 \frac{\partial^2 u}{\partial x^2} + 40\frac{\partial^2 u}{\partial y^2} = 120x$ (ii) $10\frac{\partial u}{\partial t} = 3x^4 \frac{\partial^2 u}{\partial x^2}$ (iii) $\frac{\partial^2 u}{\partial x^2} + y\frac{\partial^2 u}{\partial y^2} = 100$ (iv) $\frac{\partial u}{\partial t} + 2\frac{\partial^2 u}{\partial t \partial x} = \frac{\partial^2 u}{\partial x^2}$ [10 marks]

(b) Let $y = y(x)$, $u = u(x,t)$ be real valued smooth continuous differentiable functions

(i)Derive the finite difference approximations to the derivatives $\frac{dy}{dt}$, $\frac{\partial u}{\partial t}$, $\frac{\partial^2 u}{\partial x^2}$ [6 marks]

(ii) Give the leading truncation error term in each case. [4 marks]

Question 3 [20marks]

(a) Construct an explicit finite-difference scheme as applied to the nonhomogenous parabolic equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + 12, \qquad 0 \le x \le 1, \qquad 0 < t < 0.5$$

subject to $u(0,t) = u(1,t) = 0 \quad 0 < t$

and $u(x,0) = x(1-x) + \sin \pi x \quad 0 < x < 10$. [5 marks]

(b) Obtain a molecular formula for problem (a) above applied to the solution grid over region $W = \{(x,t): 0 \le x \le 10, 0 \le t\}$ with; $h = \Delta x = 2$; $k = \Delta t = 0.001$.

State the stability condition for the molecular formula employed. And hence compute, the numerical solutions $U_{i,j}$; for the three time levels $j = 0, 1, 2$. [15 marks]

Question 4 [20marks]

Given the wave equation

$$\frac{\partial^2 u}{\partial t^2} = 400\frac{\partial^2 u}{\partial x^2}, \qquad 0 < x < 1, \quad t > 0$$

subject to boundary conditions $u(0,t) = u(1,t) = 0 \quad$ for $t > 0$

and initial conditions $u(x,0) = \sin(2\pi x)$, $u_t(x,0) = 0$, $0 < x < 1$, $t = 0$. $h = \Delta x$, $t = 0$.

(a) Construct the explicit finite difference scheme to it. [8 marks]

(b) State the stability of the explicit finite difference scheme in part (i) above. [2 marks]

(c) Compute the approximations $U_{i,j}$; $j = 1(\mathit{first-time\ level})$, $i = 0,1,2,3,4,5.$ to the exact solutions $u(x_i, t_j)$ using $h = \Delta x = 0.2,\ k = \Delta t = 0.05\ h = \Delta x$. [10 marks]

Question 5 [20marks]

On the square $D = \{(x, y) : 0 \le x \le 2, 0 \le y \le 2\}$ consider the Dirichlet problem for the Poisson's equation,

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 10 \ \ in \ D$$

$$u = g(x, y) \ on \ S$$

(a) Use finite difference method with equal mesh spacing $h = \Delta x = \Delta y = \frac{1}{4}$, defined on D to discretize the Dirichlet problem, assuming. $g(x, y) = 0 \ on \ S$. [15 marks]

(b)Show that difference scheme takes the form $A\underline{U} = \underline{B}$: $A_{9\times9}$ *real*, *symmetric matrix*.
Deduce that the numerically computed solution $\underline{U}$ is unique [5 marks]

Question 6 [20marks]

Consider the two-dimensional diffusion equation

$$\frac{\partial u}{\partial t} = 40\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right], \qquad 0 \le x \le 4\ , 0 \le y \le 4,\ t > 0$$

subject to $u(0, y, t) = u(4, y, t) = u(x, 0, t) = u(x, 4, t) = 20$ for $t > 0$

and $u(x, y, 0) = x + y$ at $t = 0$.

(i) Construct alternate implicit finite difference schemes to it. [16 marks]

(ii) Determine the stability of the alternate implicit finite difference schemes in part (i) above. [4 marks]