(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2016/2017 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER

SPECIAL/SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE (MATHEMATICS)

COURSE CODE: MAT 427

COURSE TITLE: NUMERICAL ANALYSIS III

DATE: 13/09/17 **TIME:** 11.30 AM -1.30 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION ONE (30 MARKS)

a. Evaluate the solution of the differential equation $\frac{dy}{dx} = y^2 + 1$ by taking five terms of the Maclaurin series for $x = 0, 0.2, 0.4$ & 0.6, given $y(0) = 0$. Compare your results with exact solutions. (11 marks)

b. Find the solution of the two dimensional heat conduction equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$$

Subject to the initial conditions $u = 0$ on the boundaries conditions, $t \geq 0$ using the explicit method

$$u_{l,m}^{n+1} = (1 - 4\lambda)u_{l,m}^{n} + \lambda(u_{l-1,m}^{n} + u_{l+1,m}^{n} + u_{l,m-1}^{n} + u_{l,m+1}^{n})$$

with$h = \frac{1}{3}$ and $\lambda = \frac{1}{8}$. Integrate up to two time level. (12 marks)

c. Solve $\frac{dy}{dx} = x^2 + y^2, y(0) = 1$ by Picard's method, get the values of $y(0.2)$and $y(0.4)$. Integrate to two time level (7 marks)

QUESTION TWO (20 MARKS)

a. Find the Jacobian Matrix for the system of equations

$$f_1(x, y) = x^2 + y^2 - x^3 = 0$$
$$f_2(x, y) = x^2 - y^2 - y^3 = 0$$

At the point (1,1) using the methods:

$$\left(\frac{\partial f}{\partial x}\right)_{(x_i, y_j)} = \frac{f_{i+1,j} - f_{i-1,j}}{2h}$$

$$\left(\frac{\partial f}{\partial x}\right)_{(x_i, y_i)} = \frac{f_{i,j+1} - f_{i,j-1}}{2h}$$

With $h = k = 1$. (6 marks)

b. Solve the heat equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

Subject to the initial and boundary conditions:

$$u(x, 0) = \sin \pi x, 0 \leq x \leq 1$$

$$u(0, t) = u(1, t) = 0$$

Using the following methods:

i. The Schmist method (7 marks)

ii. The Crank-Nicolson method (7 marks)

QUESTION THREE (20 MARKS)

Solve the Initial value problem

$$\dot{u} = 4tu^3, u(0) = 1$$

With $h = 0.25$ over the interval [0,1]. Use the forth order Classical Runge-Kutta method.(20 marks)

QUESTION FOUR (20 MARKS)

a. Evaluate the integral

$$I = \int_0^1 \frac{dx}{1+x^2}$$

Using

i. Composite Trapezoidal rule (10 marks)
ii. Composite Simpson's rule (10 marks)

QUESTION FIVE (20 MARKS)

Evaluate the integral

$$I = \int_{-1}^{1} (1-x^2)^{\frac{3}{2}} cos\ x dx$$

Using

i. Gauss-Legendre three-point formula (10 marks)
ii. Gauss-Chebyshev three-point formula (10 marks)