(*Knowledge for Development*)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER

SPECIAL/SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE (MATHEMATICS)

COURSE CODE: **MAT 427**

COURSE TITLE: NUMERICAL ANALYSIS III

DATE: 03/10/18 **TIME:** 11.30 AM -1.30 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

a. Obtain a general solution of the system of equations

$$\frac{du_1}{dt} = -5u_1 + 2u_2 + t$$

$$\frac{du_2}{dt} = 2u_1 - 2u_2 + e^{-t}$$ (11Mks)

b. Find the solution of the two dimensional heat conduction equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$$

Subject to the initial conditions $u = 0$ on the boundaries conditions, $t \geq 0$ using the explicit method

$$u_{l,m}^{n+1} = (1 - 4\lambda)u_{l,m}^{n} + \lambda(u_{l-1,m}^{n} + u_{l+1,m}^{n} + u_{l,m-1}^{n} + u_{l,m+1}^{n})$$

with$h = \frac{1}{3}$ and $\lambda = \frac{1}{8}$. Integrate up to two time level. (12 marks)

c. Convert the following 2nd order initial value problem into a system of 1st order initial value problem.

$ty'' - y' + 4t^3y = 0,\ y(1) = 1, y'(1) = 2$ (7 marks)

QUESTION TWO (20 MARKS)

Evaluate the integral

$$I = \int_{-1}^{1} (1 - x^2)^{\frac{3}{2}} \cos x dx$$

Using

i. Gauss-Legendre three-point formula (10 marks)

ii. Gauss-Chebyshev three-point formula (10 marks)

QUESTION THREE (20 MARKS)

Evaluate the integral

$$I = \int_{0}^{1} \frac{dx}{1 + x^2}$$

Using

i. Composite Trapezoidal rule (10 marks)

ii. Composite Simpson's rule (10 marks)

QUESTION FOUR (20 MARKS)

Solve the Initial value problem

$$\dot{u} = 4tu^3, u(0) = 1$$

With $h = 0.25$ over the interval $[0,1]$. Use the forth order Classical Runge-Kutta method. (20 marks)

QUESTION FIVE (20 MARKS)

a. Solve $\frac{dy}{dx} = x^2 + y^2, y(0) = 1$ by Picard's method, get the values of $y(0.2)$and $y(0.4)$.
   Integrate to two time level (6 marks)

b. Solve the heat equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

Subject to the initial and boundary conditions:

$$u(x, 0) = \sin \pi x , 0 \leq x \leq 1$$

$$u(0, t) = u(1, t) = 0$$

Using the following methods:

i. The Schmist method (7 marks)

ii. The Crank-Nicolson method (7 marks)