(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

(MATHEMATICS)

COURSE CODE: MAT 427

COURSE TITLE: NUMERICAL ANALYSIS III

DATE: 18/12/17 **TIME:** 3 PM -5 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE

(a) Given the general form of an ordinary differential equation

$\emptyset(t, y, y', y'', - - - - y^{(m)}) = 0$

(i) Distinguish between m and t (2Mks)

(ii) Define a canonical representation of a differential equation. (2Mks)

(b) What is an initial value problem (2Mks)

(c) Convert the following 2nd order initial value problem into a system of 1st order initial value problem.

$ty'' - y' + 4t^3y = 0,\ y(1) = 1, y'(1) = 2$ (8Mks)

(d) Find the solution of the system of equations $\frac{du}{dt} = Au$ where $u = [u_1, u_2]^T$ and $A = \begin{bmatrix} -3 & 4 \\ -2 & 3 \end{bmatrix}$ (9Mks)

(e) Show that for any u_0, the initial value problem $\frac{du}{dt} = f(t, u), u(t_0) = \eta_0$ has a unique solution u(t) for $t \in [t_0, b]$ hence define the Lipschitz constant. (7Mks)

QUESTION TWO

(a) Define

(i) truncation error (2Mks

(ii) convergence (2Mks

(iii) Euler method (2Mks)

(b) Obtain a general solution of the system of equations

$\frac{du_1}{dt} = -5u_1 + 2u_2 + t$

$\frac{du_2}{dt} = 2u_1 - 2u_2 + e^{-t}$ (14Mks)

QUESTION THREE

(a) Use the Euler method to solve numerically the initial value problem

$u^2 = -2tu,\ u(0) = 1$

With $h = 0.2$ 0.1 and 0.05 on the interval [0.1] (16Mks)

(b) Neglecting round-off errors determine the bound for the error in (a) above (14Mks)

QUESTION FOUR

(a) what is a homogenous Boundary value problem (2Mks)

(b) distinguish between an Eigenvalue and Eigen function of a boundary value problem. (3Mks)

(c) State any three conditions satisfied for a unique and existing solution of

$Bv\rho u'' = f(x_1 u_x u^1), x \in (a, b)$ (3Mks)

(d) for the analysis of the numerical solution of the test equation $u^1 = \lambda u$. Show that the propagation factor should satisfy the condition $|E(\lambda h)| <|$ (12MKS)

QUESTION FIVE

(a) find the solution of the initial value problem $\frac{du}{dt} = Au, u(0) = [1,0]$ where $A = \begin{bmatrix} -2 & 1 \\ 1 & -20 \end{bmatrix}$

(16Mks)

(b) is the system in (a) above asymptotically stable, explain (4Mks)