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(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2016/2017 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: MAT 401

COURSE TITLE: TOPOLOGY I

DATE: 11/09/17

TIME: 3 PM -5 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION 1 (30 MARKS)

- a) Define the following terms: interior point, closed set, boundary point, closure point and accumulation point. (5 marks)
- b) The intersection $N \cap M$ of any two neighbourhoods N and M of a point p is also a neighbourhood of p . Prove. (5 marks)
- c) The class $\tau_1 = \{X, \emptyset, \{a\}, \{a, b\}, \{a, c, d\}, \{a, b, c, d\}, \{a, b, e\}\}$ is a topology on $X = \{a, b, c, d, e\}$. Find A' , the derived set of $A = \{c, d, e\} \subset X$. (6 marks)
- d) Define a homeomorphism. (2 marks)
- e) Let $X = \{1, 2, 3\}$. Show that $\beta = \{\{1, 2\}, \{2, 3\}\}$ cannot be a base for any topology X . (6 marks)
- f) If $A \subset B$, then $\bar{A} \subset \bar{B}$. Prove. (6 marks)

QUESTION 2 (20 MARKS)

- a) Define a topological space. (3 marks)
- b) Let $\tau_1 = \{X, \emptyset, \{a\}\}$ and $\tau_2 = \{X, \emptyset, \{b\}\}$ be topologies on $X = \{a, b, c\}$. Show whether the union $\tau_1 \cup \tau_2$ is a topology X or not? (9 marks)
- c) Prove that if $A \subset B$, then every limit point of A is a limit point B . (8 marks)

QUESTION 3 (20 MARKS)

- a) Define a Hausdorff space. (3 marks)
- b) Prove that all metric spaces are Hausdorff spaces. (10 marks)
- c) Consider the following topology on $X = \{a, b, c, d, e\}$: $\tau = \{X, \emptyset, \{a\}, \{a, b\}, \{a, c, d\}, \{a, b, c, d\}, \{a, b, e\}\}$. Determine the closure of the sets $\{a\}$, $\{b\}$ and $\{c, e\}$ (7 marks)

QUESTION 4 (20 MARKS)

- a) Define continuity of a function between topological spaces. (2 marks)
- b) Let X, Y, Z be topological spaces, and let $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ be continuous functions. Prove that the composition $g \circ f: X \rightarrow Z$ of the functions f and g is continuous. (9 marks)
- c) Let $\{\tau_i\}$ be a collection of topologies on a set X . If a function $f: X \rightarrow Y$ is continuous with respect to each τ_i , prove that f is continuous with respect to the intersection topology $\tau = \bigcap_i \tau_i$. (9 marks)

QUESTION 5 (20 MARKS)

- a) Let $\tau_1 = \{X, \emptyset, \{a\}, \{a, b\}, \{a, c, d\}, \{a, b, c, d\}, \{a, b, e\}\}$ be a topology on $X = \{a, b, c, d, e\}$. Let $A = \{a, b, c\} \subset X$. Find
- $\text{Int}(A)$, the interior of A . (4 marks)
 - $\text{Ent}(A)$, the exterior of A . (4 marks)
 - $\partial(A)$, the boundary of A . (4 marks)
- b) Let A be a subset of a topological space X and \bar{A} be the closure of A . Show that $\bar{A} = \text{Int}(A) \cup \partial(A)$. (8 marks)