(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: MAT 401

COURSE TITLE: TOPOLOGY I

DATE: 20/12/17 **TIME:** 8 AM -10 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION 1 (30 MARKS)

a) Define the following terms: discrete topology, trivial topology, boundary point and cofinite topology. (4 marks)

b) Let X be a topological space. Prove that the empty set $\emptyset$ and the whole space X are closed. (4 marks)

c) The intersection $N \cap M$ of any two neighbourhoods N and M of a point p is also a neighbourhood of p. Prove. (5 marks)

d) Consider the following class of subsets of $X = \{a,b,c,d\}$. Determine whether or not $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a,b\}, \{b,d\}\}$ is a topology on X. (4 marks)

e) The class $\tau = \{X, \emptyset, \{a\}\{a,b\}, \{a,c,d\}, \{a,b,c,d\}, \{a,b,e\}\}$ is a topology on $X = \{a,b,c,d,e\}$. Find A', the derived set of $A = \{c,d,e\} \subset X$. (5 marks)

f) Define a Hausdorff space. (2 marks)

g) Let $X = \{1,2,3\}$. Show that $\beta = \{\{1,2\},\{2,3\}\}$ cannot be a base for any topology X. (6 marks)

QUESTION 2 (20 MARKS)

a) Define a topological space. (2 marks)

b) Prove that all metric spaces are Hausdorff spaces. (7 marks)

c) Prove that if $A \subset B$, then every limit point of A is a limit point B. (6 marks)

d) Let $\tau = \{X, \emptyset, \{1\}\{1,2\}, \{1,3,4\}, \{1,2,3,4\}, \{1,2,5\}\}$ be a topology on $X = \{1,2,3,4,5\}$:Find the neighbourhood system of point 5. (5 marks)

QUESTION 3 (20 MARKS)

a) Define a homeomorphism. (2 marks)

b) Consider the following topology on $X = \{1,2,3,4,5\}$: $\tau = \{X, \emptyset, \{1\}\{1,2\}, \{1,3,4\}, \{1,2,3,4\}, \{1,2,5\}\}$.

   i. Determine the closed subsets of X (2 marks)

   ii. Determine the closure of the sets $\{1\}$, $\{2\}$ and $\{3,5\}$ (5 marks)

   iii. Which sets in (ii) are dense in X? (1 mark)

c) Let $\tau_1 = \{X, \emptyset, \{a\}\}$ and $\tau_2 = \{X, \emptyset, \{b\}\}$ be topologies on $X = \{a,b,c\}$. Show whether the union $\tau_1 \cup \tau_2$ is a topology X or not? (5 marks)

d) If $A \subset B$, then $\overline{A} \subset \overline{B}$. Prove. (5 marks)

QUESTION 4 (20 MARKS)

a) Define continuity of a function between topological spaces. (2 marks)

b) Let $X; Y; Z$ be topological spaces, and let $f: X \to Y$ and $g: Y \to Z$ be continuous functions. Prove that the composition $g \circ f: X \to Z$ of the functions f and g is continuous. (9 marks)

c) Let $\{\tau_i\}$ be a collection of topologies on a set X. If a function $f: X \to Y$ is continuous with respect to each τ_i, prove that f is continuous with respect to the intersection topology $\tau = \cap_i \tau_i$. (9 marks)

QUESTION 5 (20 MARKS)

a) Let $\tau_1 = \{X, \emptyset, \{a\}\{a,b\}, \{a,c,d\}, \{a,b,c,d\}, \{a,b,e\}\}$ be a topology on $X = \{a,b,c,d,e\}$.
   Let $A = \{a,b,c\} \subset X$. Find

   i. τ_A, the relative topology on A. (3 marks)

   ii. $Int\ (A)$, the interior of A. (3 marks)

   iii. $Ent\ (A)$, the exterior of A. (3 marks)

   iv. $\partial(A)$, the boundary of A. (3 marks)

b) Let A be a subset of a topological space X and $\bar{A}$ be the closure of A. Show that $\bar{A} = Int\ (A) \cup \partial\ (A)$. (8 marks)