



15

*(Knowledge for Development)*

**KIBABII UNIVERSITY**  
**UNIVERSITY EXAMINATIONS**  
**2016/2017 ACADEMIC YEAR**  
**FOURTH YEAR FIRST SEMESTER**  
**SPECIAL/SUPPLEMENTARY EXAMINATION**  
**FOR THE DEGREE OF BACHELOR OF EDUCATION AND**  
**BACHELOR OF SCIENCE (MATHEMATICS)**

**COURSE CODE:** MAT 425

**COURSE TITLE:** FLUID MECHANICS II

**DATE:** 15/09/17

**TIME:** 11.30 AM -1.30 PM

---

**INSTRUCTIONS TO CANDIDATES**

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 4 Printed Pages. Please Turn Over.



### QUESTION ONE (30 MARKS)

- a. Define the following terms. (3 marks)
- (i) Potential flow
  - (ii) Sink
  - (iii) Source
- b. A source of strength  $10\text{m}^2/\text{s}$  is located at  $(-1, 0)$  and a sink of strength  $20\text{m}^2/\text{s}$  is located at  $(1, 0)$ . Find the velocity and stream function at  $P(1,1)$ . If the dynamic pressure at infinity is zero for density of  $2\text{Kg}/\text{m}^3$ , calculate the dynamic pressure at  $P$ . (5 Marks)
- c. The resistance force  $R$  of a supersonic plane during flight can be considered as dependent upon the length of the aircraft  $l$ , velocity  $v$ , air viscosity  $\mu$ , air density  $\rho$  and bulk modulus of air  $k$ . Find an expression for  $R$ . (6 marks)
- d. Show that for a two dimensionally axially symmetric boundary layer flow.

$$\int_0^\infty \left(1 - \frac{u}{U}\right)^2 \frac{r}{a} dn - \delta_1 - \delta_2, \int_0^\infty \left(1 - \frac{u}{U}\right)^3 \frac{r}{a} dn - \delta_1 - 3\delta_2 + \delta_3$$

Where  $n$  the normal distance from the surface of the body is,  $r$  is the axial distance and  $a$  is the reference radius which maybe a function of the axial distance. (5 marks)

- e. Determine the nature of the flow pattern represented by the following potential function

$$W = \frac{a}{z} \quad (5 \text{ Marks})$$

- f. The velocity potential function for a two dimensional flow is  $\Phi = x(2y - 1)$ . At a point  $P(4, 5)$  determine;

- i) The velocity (3 marks)
- ii) The value of the stream function (4 marks)

### QUESTION TWO (20 MARKS)

- a. Show that for an incompressible steady flow with constant velocity components

$$u(y) = y \frac{U}{h} + \frac{h^2}{2\mu} \left(-\frac{dp}{dx}\right) \frac{y}{h} \left(1 - \frac{y}{h}\right)$$

$$v = w = 0$$

Satisfy the equation of motion, when the body force is neglected  $h, U, \frac{dp}{dx}$  are constants and  $P = P(x)$ . (6 marks)



- b. A source and a sink of equal strength are placed at the points  $(\pm \frac{1}{2}a, 0)$  within a fixed circular boundary  $y^2 + x^2 = a^2$ . Show that the streamlines are given by

$$(r^2 - \frac{1}{4}a^2)(r^2 - 4a^2) - 4a^2y^2 = ky(r^2 - a^2). \quad (5 \text{ marks})$$

- c. In the region bounded by a fixed quadrantal arc and its radii, deduce the motion due to a source and an equal sink situated at the ends of one of the bounding radii. Show that the stream line leaving either end at an angle  $\alpha$  with the radius is

$$r^2 \sin(\alpha + \theta) = x^2 \sin(\alpha - \theta). \quad (6 \text{ marks})$$

- d. A vortex of circulation  $2\pi k$  is at the point  $z = na$  ( $n > 1$ ) in the presence of a plane circular boundary  $Z = a$ , around which there is a circulation  $2\pi \lambda k$ . Show that

$$\lambda = \frac{1}{(n^2 - 1)} \quad (6 \text{ marks})$$

### QUESTION THREE (20 MARKS)

- a. Find the equations of the streamlines due to uniform line source of strength  $m$  through the points A(-c,0) B(c,0) and uniform line sink of strength  $2m$  through the origin (5 Marks)
- b. Show that the velocity vector  $\mathbf{q}$  is everywhere tangent to the line in the  $xy$  plane along which  $\psi(x, y) = \text{constant}$ . (3 marks)
- c. Discuss the flow for which  $W = z^2$ . (4 marks)
- d. Determine the stream function  $\psi(x, y, t)$  for the given velocity field given by  $u = Ut, V = x$ . (3 marks)
- e. Discuss the flow due to a uniform line doublet at 0 of strength  $\mu$  per unit length and its axis being along OX. (5 marks)

### QUESTION FOUR (20 MARKS)

- a. State the Buckingham's  $\pi$ -theorem. (2 marks)
- b. Differentiate between the following terms as used in dimensional analysis (6 marks)
- Geometrical symmetry
  - Kinematic similarity
  - Dynamic similarity



- c. The pressure difference  $\Delta P$  in a pipe of diameter  $D$  and length  $l$  due to turbulent flow depends on the velocity  $V$ , viscosity  $\mu$ , density  $\rho$ , roughness  $K$ . Using Buckingham's  $\pi$ -theorem, obtain an expression for  $\Delta P$ . (7 marks)
- d. Using Rayleigh's technique, show that the resistance ( $R$ ) to the motion of a sphere of diameter ( $D$ ) moving with a uniform velocity ( $V$ ) through a real fluid having density  $\rho$  and viscosity  $\mu$  is given by  $R = \rho D^2 V^2 f\left(\frac{\mu}{\rho V D}\right)$ . (5 marks)

**QUESTION FOUR (20 MARKS)**

- a) State the theorem of Blasius. (2 marks)
- b) Verify that  $W = iK \log \left\{ \frac{z-ia}{z+ia} \right\}$  is the complex potential of a steady flow of liquid about a cylinder the plane  $y = 0$  being a rigid boundary. Find the forces exerted by the liquid on unit length of the cylinder. (7 marks)
- c) Find the image of a line source in a circular cylinder. (5 marks)
- d) Show that the velocity potential  $\phi = \frac{c}{2} (x^2 + y^2 - 2z^2)$  satisfies the Laplace equation and determine its streamlines. (6 marks)