(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER

SPECIAL/SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE (MATHEMATICS)

COURSE CODE: **MAT 425**

COURSE TITLE: FLUID MECHANICS II

DATE: 05/10/18 **TIME:** 11.30 AM -1.30 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

a. Find the equations of the streamlines due to uniform line source of strength m through the points A(-c, 0), B(c, 0) and a uniform line sink of strength $2m$ through the origin. (6 marks)

b. The velocity potential function for a two dimensional flow is $\Phi = x(2y - 1)$. At a point P(4, 5) determine;

   i) The velocity (3 marks)

   ii) The value of the stream function (4 marks)

c. The resistance force R of a supersonic plane during flight can be considered as dependent upon the length of the aircraft l, velocity v, air viscosity μ, air density ρ and bulk modulus of air k. Find an expression for R. (6 marks)

d. Show that for an incompressible steady flow with constant velocity components

$$u(y) = y\frac{U}{h} + \frac{h^2}{2\mu}\left(-\frac{dp}{dx}\right)\frac{y}{h}\left(1 - \frac{y}{h}\right)$$

$$v = w = 0$$

Satisfy the equation of motion, when the body force is neglected $h, U, \frac{dp}{dx}$ are constants and $P = P(x)$. (6 marks)

e. Show that for a two dimensionally axially symmetric boundary layer flow.

$$\int_0^{\infty}(1 - \tfrac{u}{U})^2\tfrac{r}{a}\,dn = \delta_1 - \delta_2,\ \int_0^{\infty}(1 - \tfrac{u}{U})^3\tfrac{r}{a}\,dn = \delta_1 - 3\delta_2 + \delta_3$$

Where n the normal distance from the surface of the body is, r is the axial distance and a is the reference radius which maybe a function of the axial distance. (5 marks)

QUESTION TWO (20 MARKS)

a. State the Buckingham's π-theorem. (2 marks)

b. Differentiate between the following terms as used in dimensional analysis (6 marks)

   i) Geometrical symmetry

   ii) Kinematic similarity

   iii) Dynamic similarity

c. The pressure difference ΔP in a pipe of diameter D and length l due to turbulent flow depends on the velocity V, viscosity μ, density ρ, roughness K. Using Buckingham's π-theorem, obtain an expression for ΔP. (7 marks)

d. Using Rayleigh's technique, show that the resistance (R) to the motion of a sphere of diameter (D) moving with a uniform velocity (V) through a real fluid having density ρ and viscosity μ is given by $R = \rho D^2 V^2 f\left(\frac{\mu}{\rho V D}\right)$. (5 marks)

QUESTION THREE (20 MARKS)

a. Define the following terms
   i. Sink
   ii. Source
   iii. Doublet (3 marks)

b. Show that the velocity vector **q** is everywhere tangent to the line in the xy plane along which $\psi(x, y) = constant$. (3 marks)

c. Discuss the flow for which $W = z^2$. (5 marks)

d. Determine the stream function $\psi(x, y, t)$ for the given velocity field given by $u = Ut, V = x$. (4 marks)

e. Discuss the flow due to a uniform line doublet at 0 of strength μ per unit length and its axis being along OX. (5 marks)

QUESTION FOUR (20 MARKS)

a) State Milne Thomson's circle theorem. (2 marks)

b) A source and a sink of equal strength are placed at the points $\left(\pm\frac{1}{2}a, 0\right)$ within a fixed circular boundary $y^2 + x^2 = a^2$. Show that the streamlines are given by
$(r^2 - \frac{1}{4}a^2)(r^2 - 4a^2) - 4a^2y^2 = ky(r^2 - a^2)$. (5 marks)

c) In the region bounded by a fixed quadrantal arc and its radii, deduce the motion due to a source and an equal sink situated at the ends of one of the bounding radii. Show that the stream line leaving either end at an angle α with the radius is
$r^2 \sin(\alpha + \theta) = x^2 \sin(\alpha - \theta)$. (6 marks)

d) A vortex of circulation $2\pi k$ is at the point $z = na \ (n > 1)$ in the presence of a plane circular boundary $Z = a$, around which there is a circulation $2\pi \lambda k$. Show that
$\lambda = \frac{1}{(n^2 - 1)}$ (6 marks)

QUESTION FIVE (20 MARKS)

a) State the theorem of Blasius. (2 marks)

b) Verify that $W = iK\, log\,\{\frac{z-ia}{z+ia}\}$ is the complex potential of a steady flow of liquid about a cylinder the plane $y = 0$ being a rigid boundary. Find the forces exerted by the liquid on unit length of the cylinder. (7 marks)

c) Find the image of a line source in a circular cylinder. (5 marks)

d) Show that the velocity potential $\phi = \frac{a}{2}\,(x^2 + y^2 - 2z^2)$ satisfies the Laplace equation and determine its streamlines. (6 marks)