(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

(MATHEMATICS)

COURSE CODE: MAT 425

COURSE TITLE: FLUID MECHANICS II

DATE: 20/12/17 **TIME:** 8 AM -10 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

a) Define the following terms (4 Marks)

(i) Potential flow

(ii) Source

(iii) Sink

(iv) Doublet

b) State the theorem of Blasius (2 Marks)

c) A stream function is given by

$$\psi = 3x^2 - y^3$$

Determine the magnitude of the velocity at the point (3,1) (4 Marks)

d) Discuss the flow determined by $W = z^2$ (7 Marks)

e) A source and a sink of equal strength are placed at the points $\left(\pm\frac{1}{2}a, 0\right)$ within a fixed circular boundary $y^2 + x^2 = a^2$. Show that the streamlines are given by

$$(r^2 - \tfrac{1}{4}a^2)(r^2 - 4a^2) - 4a^2 y^2 = ky(r^2 - a^2).$$ (6 marks)

f) The pressure difference ΔP in a pipe of diameter D and length l due to turbulent flow depends on the velocity V, viscosity μ, density ρ, roughness K. Using Buckingham's π-theorem, obtain an expression for ΔP. (7 marks)

QUESTION TWO (20 MARKS)

a) Differentiate between the following terms as used in dimensional analysis (6 marks)

(i) Geometrical symmetry

(ii) Kinematic similarity

(iii) Dynamic similarity

b) Show that for a two dimensionally axially symmetric boundary layer flow.

$$\int_0^\infty \left(1 - \frac{u}{U}\right)^2 \frac{r}{a}\, dn = \delta_1 - \delta_2, \int_0^\infty \left(1 - \frac{u}{U}\right)^3 \frac{r}{a}\, dn = \delta_1 - 3\delta_2 + \delta_3$$

Where n the normal distance from the surface of the body is, r is the axial distance and a is the reference radius which maybe a function of the axial distance. (4 marks)

c) A velocity is given by $\boldsymbol{q} = -x\boldsymbol{i} + (y + t)\boldsymbol{j}$. Find the stream function and streamlines for this field at t=2 (4 Marks)

d) A two dimensional flow field is given by $\psi = xy$

(i) Show that the flow is irrotational (2 Marks)

(ii) Find the velocity potential (3 Marks)

(iii) Find the streamlines and potential lines (1 Marks)

QUESTION THREE (20 MARKS)

a) What arrangement of sources and sinks will give rise to the function $W = \log\left(z - \frac{a^2}{z}\right)$ and prove that two streamlines subdivide into the circle r=a and axis of y (5 Marks)

b) Find the equations of the streamlines due to uniform line source of strength *m* through the points A(-c,0) B(c,0) and uniform line sink of strength *2m* through the origin (5 Marks)

c) Discuss the flow due to a uniform line doublet at 0 of strength μ per unit length and its axis being along OX. (5 marks)

d) A flow field is described by the equation $\psi = y - x^2$

(i) Sketch the streamline $\psi = 0, \psi = 1,\ and\ \psi = 2$ (2 Marks)

(ii) Derive an expression for the velocity V at any point in the flow field (3 Marks)

QUESTION FOUR (20 MARKS)

a) By use of a well labeled diagram, define what is meant by boundary layer as used in fluid dynamics (3 Marks)

b) Find the displacement thickness, the momentum thickness, and energy thickness for the velocity distribution in the boundary layer given by $\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$ (6 Marks)

c) Verify that $W = ik \log\left\{\frac{z - ia}{z + ia}\right\}$ is the complex potential of a steady flow of liquid about a cylinder with the plane $y = 0$ being the rigid boundary. Find the forces exerted by the liquid on unit length of the cylinder. (6 Marks)

d) Find the image of a line source in a circular cylinder (5 Marks)

QUESTION FIVE (20 MARKS)

a) State the Buckingham's π-theorem. (2 marks)

b) Show that the velocity vector **q** is everywhere tangent to the line in the xy plane along which $\psi(x, y) = constant$. (5 marks)

c) Using Rayleigh's technique, show that the resistance (R) to the motion of a sphere of diameter (D) moving with a uniform velocity (V) through a real fluid having density ρ and viscosity μ is given by $R = \rho D^2 V^2 f\left(\frac{\mu}{\rho V D}\right)$. (6 marks)

d) Show that for an incompressible steady flow with constant velocity components

$$u(y) = y\frac{U}{h} + \frac{h^2}{2\mu}\left(-\frac{dp}{dx}\right)\frac{y}{h}\left(1 - \frac{y}{h}\right)$$

$$v = w = 0$$

Satisfy the equation of motion, when the body force is neglected $h, U, \frac{dp}{dx}$ are constants and $P = P(x)$. (7 marks)