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(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2016/2017 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: MAT 325

COURSE TITLE: FLUID MECHANICS I

DATE: 14/09/17 **TIME:** 8 AM -10 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 4 Printed Pages. Please Turn Over.

QUESTION ONE (30 MARKS)

a) Differentiate between the following fluid flow motion (4 Marks)

(i) Lagrangian

(ii) Eulerian

b) The pressure distribution in a non viscous fluid is given by $P = 2xy + z^2 + 10$. Find the force per unit volume on an element of the fluid in the direction $2i - 3j + \sqrt{3}k$ at the point (1, 0, 5) (5 Marks)

c) Given the velocity field $v = (16x + y)i + 10j + xz\ k$. Determine the angular velocity of a fluid element at $r = 6i + 3j + 2k$ (5 Marks)

d) Define the following terms

(i) Streamlines

(ii) Pathlines

(iii) Stream function

Explain the condition under which (i) and (ii) above maybe the same (6 Marks)

e) Examine whether the velocity components given by

$$u = -4ax(x^2 - 3y^2)$$

$$v = 4ay(3x^2 - y^2)$$

Represent a physically two dimensional flow, if so whether the flow is rotational or irrotational (5 marks)

f) A three dimensional flow is given by $u = yz + t, \quad v = xz - t, \quad w = xy$. Find the velocity and acceleration at P(1,2,3) after 1 sec (5 Marks)

QUESTION TWO (20 MARKS)

a) (i) State Kelvin's circulation theorem (2 Marks)

(ii) Calculate the circulation around a unit circle with centre at the origin given that the steady velocity field is (5 Marks)

$$V = \frac{-yi + xj}{x^2 + y^2}$$

b) Determine the density, specific gravity, and mass of air in a room whose dimensions are 4m×5m×6m at 100kPa and 25^0c. (Assume the ideal gas constant of air is R=0.287kPa m^3/kg) (5 Marks)

c) The velocity of a fluid in spherical co-ordinates is given by

$$v = \frac{m}{r^2} e_r$$

Where m is a constant. Show that if S is a closed surface centred at the origin with radius r, then the volume flux is $4\pi m$. (4 Marks)

d) A ball initially at rest at x=0 in a viscous fluid is pulled in a straight line by a string. A time dependent force $F_a(t)$ is applied to the string which causes the ball to move according to $x(t) = \frac{L}{2}[1 - \cos\left(\frac{\pi t}{T}\right)]$.

At time $t = T$, the ball comes to rest at x=L and the force is removed. As the ball moves through the fluid it experiences a dragging force proportional to its speed $F_d = -c\dot{x}(t)$. How much work is done by the applied force to move the ball from x=0 to x=L?

(4 Marks)

QUESTION THREE (20 MARKS)

a) Show that if a fluid is isothermal perfect gas, then

$$P = P_1 \exp(\frac{-g}{RT}(z - z_1))$$

Where P_1 is pressure at $z = z_1$ (Note that isothermal means constant temperature, perfect gas obeys $P = \rho RT$, where ρ=density, T=absolute temperature, R=Constant) (6 Marks)

b) Write down Bernoullis equation for steady, ideal, irrotational flow in a gravitational field. Water enters one end of horizontal pipe at a speed of 4m/s and leaves the other end at 10m/s. Calculate the pressure difference across two ends of the pipe (9 Marks)

c) Describe the continuum hypothesis as used in fluid mechanics (5 Marks)

QUESTION FOUR (20 MARKS)

a) State the three laws of thermodynamics (3 Marks)

b) Derive the equation of the hyrostatic equilibrium of a static fluid in the form

$\nabla P = \rho f$

Where P is the pressure, ρ is density of the fluid and f is the body force per unit mass (7 Marks)

c) Show that if gravitational force is the only body force then the relation above takes the form $\frac{dP}{dz} = -\rho g$ (5 Marks)

d) The density of a fluid is given by $\rho = xyz$ with velocity $v = yi + zj + xk$. Show that

$$y^2z + z^2x + x^2y = 0$$ (5 Marks)

QUESTION FIVE (20 MARKS)

a) Explain the term barotrophic flow (2 Marks)

b) Define the following terms (2 Marks)

(i) Incompressible flow

(ii) Vorticity

c) A fluid has velocity given by

$v = (xy^2 + tx)i + (yt^2 + x^2)j + (2t^3 + y^2z)k$

(i) Show that the vortex field is steady (3 Marks)

(ii) Find the angular velocity of fluid particle centered at (1, 2, -2) (3 Marks)

d) Show that for a frictionless fluid, if once irrotational will always be so and is also true when each particle is acted on by a resistance varying as the velocity (5 Marks)

e) State and prove the equation governing kinetic energy of a liquid at infinity (5 Marks)