(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2016/2017 ACADEMIC YEAR

THIRD YEAR SECOND SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: **MAT 324**

COURSE TITLE: **NUMERICAL ANALYSIS II**

DATE: 18/09/17 **TIME:** **11.30 AM -1.30 PM**

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

Question 1 **[30 Marks]**

(a) Solve the following system of equations using Crout's method

$$\begin{bmatrix} 2x_1 + x_2 + x_3 \\ x_1 + 2x_1 + x_3 \\ x_1 + x_2 + 2x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$$

[6 Marks]

(b) Determine numerically the dominant eigen value and the corresponding eigen vector for the following matrix using power method

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

. **[6 Marks]**

(c) By use of Picard's method for successive approximation. compute the value of y when x= 0.25 given y(0)=1 and $\frac{dy}{dx} = 3x + y^2$ **[6 Marks]**

(d) Obtain the cubic spline approximation for the following data

x	0	1	2
y =f(x)	-1	4	27

with $M_0 = M_2 = 0$ Hence interpolate x=0.5 **[6 Marks]**

(e) Evaluate the integral

$$I = \int_0^2 \frac{x^2 + 2x + 1}{1 + (x + 1)^4}$$

Using Gauss three point formula **[6 Marks]**

(f) Use Jacobi iterative scheme to obtain solution of the system of equations correct to 2dp

$$\begin{bmatrix} 5 & -2 & 1 \\ 1 & 4 & 2 \\ 1 & 2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 17 \end{bmatrix}$$

[7 Marks]

Question 2 **[20 Marks]**

(a) Solve the following system of equations by relation technique

$$\begin{bmatrix} 8x_1 + x_2 - x_3 \\ 2x_1 + x_2 + 9x_3 \\ x_1 - 7x_2 + 2x_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 12 \\ -4 \end{bmatrix}$$

take (0,0,0) as initial condition **[10 Marks]**

(b) Find all the eigen values of tridiagonal symmetric matrix X which lie in the interval (0,5)

$$X = \begin{bmatrix} 2 & 4 & 0 & 0 \\ 4 & 10 & 3 & 0 \\ 0 & 3 & 9 & -1 \\ 0 & 0 & -1 & 5 \end{bmatrix}$$

[10 Marks]

Question 3 **[20 Marks]**

(a) Use second order Taylor series method (2,3) to compute the solution to the initial value problem

$$\frac{dy}{dx} = -xy^2, \quad y(2) = 1, \quad \text{take } h = 0.1$$

[10 Marks]

(b) by use of Thomas Algorithm, solve the following set of tridiagonal algebraic equation

$$\begin{aligned} 3x_1 - x_2 &= 5 \\ 2x_1 - 3x_2 + 2x_3 &= 5 \\ x_2 + 2x_3 + 5x_4 &= 10 \\ x_3 - x_4 &= 1 \end{aligned}$$

[10 Marks]

Question 4 **[20 Marks]**

(a) Construct the least squares quadrate approximation function

$$y = e^x \quad [0, 1]$$

[10 Marks]

(b) Find the eigen values and eigen vectors for the matrix A

$$A = \left\{ \begin{matrix} 5 & 3 \\ 3 & 5 \end{matrix} \right\}$$

[4 Marks]

(c) Obtain the cubic spline approximation for the following data

x	0	1	2	3
y =f(x)	1	2	33	244

with $M_0 = M_3 = 0$ hence interpolate at x=2.5 **[4 Marks]**

Question 5 **[20 Marks]**

(a) Given the diagonal symmetric matrix

$$C = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & -1 \end{pmatrix}$$

Use Gershgorin theorem to locate the interval $I = (\alpha, \beta)$ for which $\lambda \in I$ the eigen values of C **[6 Marks]**

(b) Use Characteristic polynomial to determine the matrix inverse of A

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 3 & 4 \\ 5 & 6 & -2 \end{bmatrix}$$

[6 Marks]

(c) Evaluate the integral

$$I = \int_0^1 \frac{dx}{1 + x}$$

using Gauss a three point formula **[7 Marks]**

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