(*Knowledge for Development)*

KIBABII UNIVERSITY

MAIN EXAMINATION

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS AND EDUCATION SCIENCE/ARTS (SB)

COURSE CODE: **MAT 321**

COURSE TITLE: ODE I

DATE: **07/12/2018** **TIME:** **3.00-5.00pm**

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of **3 Printed Pages**. Please Turn Over.

QUESTION ONE (30 MARKS)

a) Solve the following

i. $\frac{dy}{dx} = 5x + cosx$ (2mark)

ii. $\frac{dy}{dx} = \frac{x^2+2}{y}$ (3marks)

b) State the degree and order of the following

$$\frac{d^4y}{dx^4} + \left(\frac{d^3y}{dx^3}\right)^2 + 3\left(\frac{d^2y}{dx^2}\right)^5 + \frac{dy}{dx} + 3y = 0$$ (2marks)

c) Show that $Ax^2 + By^2 = 1$ is the solution of $x\left(y\frac{d^2y}{dx^2} + \left[\frac{dy}{dx}\right]^2\right) - y\frac{dy}{dx} = 0$ (5marks)

d) Check for exactness and solve $(4x+3y+1)dx + (3x+2y+1)dy = 0$ (3marks)

e) Given $\frac{dy}{dx} = x + y, y(0) = 1$ estimate the value of $y(0.2)$ (5marks)

f) Solve $\frac{dy}{dx} = \frac{2y^4 + x^4}{xy^3}$ (5marks)

g) Solve $\frac{dy}{dx} = \frac{2xy}{x^2 - y^2}$ (5marks)

QUESTION TWO (20 MARKS)

a) Solve the differential equation
$\frac{d^2x}{dt^2} - 3\frac{dx}{dt} + 3x = 0$ given that at $t = 0$ $x = 0$ *and* $\frac{dx}{dt} = 3$ (5marks)

b) Solve the following linear differential equation, using a suitable integrating factor
$\frac{dT}{dt} + kT = 100k$ (5marks)

c) A person places $20,000 in a savings account which pays 5 percent interest per annum, compounded continuously. Find

i) the amount in the account for three years, and (5marks)

ii) the time required for the account to double in value, presuming no withdrawals and no additional deposits (5marks)

QUESTION THREE (20 MARKS)

a) Solve $y'+y=\sin x$ (3marks)

b) Determine whether the differential equation $\left(2x^2t-2x^3\right)dt+\left(4x^3-6x^2t+2xt^2\right)dx=0$ is exact. (2marks)

c) Solve the differential equation given in (b) above (5marks)

d) A bacteria culture is known to grow at a rate proportional to the amount present. After one hour, 1000 strands of the bacteria are observed in the culture; and after four hours, 3000 strands. Find

i) An expression for the approximate number of strands of the bacteria present in the culture at any time t (2marks)

ii) the approximate number of strands of the bacteria originally in the culture (8marks)

QUESTION FOUR (20 MARKS)

a) Find the Taylor series solution of the differential equation $\frac{dy}{dx}=2\frac{y}{x}, y(1)=2$ up to the term in $(x-1)^4$ (10 marks)

b) Given $\frac{dy}{dx}=x+y.y(0)=1$ estimate y when $x=1$using improved Eulers method. Compare the exact solution to approximated solution and calculate the percentage error. (10 marks)

QUESTION FIVE (20 MARKS)

a) Find a suitable integrating factor and solve the equation $xydx-\left(x^2+2y^2\right)dy=0$ (5marks)

b) Solve $(2x+3y-5)\frac{dy}{dx}+3x+2y-5=0$ (5marks)

c) The p.d., V ,between the plates of a capacitor C charged by a steady voltage E through a resistor R is given by the equation $CR\frac{dV}{dt}+V=E$ Solve the equation for V given that at $t=0, v=0$ (5marks)

d) The motion of a vibrating body is defined by the differential equation $\frac{d^2y}{dx^2}+10\frac{dy}{dx}+25y=x^2$ where y is the displacement. Solve the given equation using the method of undetermined coefficient (5marks)

Course Description

First order equations: Analytic Methods of solution, Applications. Equations with homogeneous coefficients. Exact Differential equations. Integrating factors and substitution methods. Linear differential equations: Homogenous and non homogenous, methods of solution for equations of constant and variable coefficients. Qualitative treatment of scalar first order equations: Slope fields, equilibrium solutions, stability, Bifurcation, Applications. First order first degree differential equations in three variables, Power series solutions of linear equations near ordinary points.