(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

THIRD YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: **MAT 321**

COURSE TITLE: ODE I

DATE: 01/08/18 **TIME:** 9 AM -11 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION 1

(a) Solve the initial value problem $y' = 3x$, $y(0) = 5$ **[3mks]**

(b) Verify that $y = cx^2$ is the solution of $xy' - 3y = 0$, hence find the particular solution given that $y(3) = 2$ **[6mks]**

(c) Show that $f(x, y) = x^2y - 4x^3 + 3xy^2$ is homogenous function of degree three **[4mks]**

(d) Solve the differential equation $\frac{dy}{dx} = \frac{x+3y}{2x}$ **[5mks]**

(e) Test for exactness in $(2xy - 3x^2)dx + (x^2 - 2y)dy = 0$. **[6mks]**

(f) Find the general solution of $\frac{dy}{dx} + 5 = 6x$, $y(1) = 2$ **[6mks]**

QUESTION 2

(a) Show that $(y^2 + ycosx)dx + (3xy + 2sinx)dy = 0$ is not exact then find an integrating factor and solve it. **[10mks]**

(b) Find the equation of a curve that passes through the point $(1, 3)$ and has a slope of $\frac{y}{x^2}$ at the point (x, y) **[7mks]**

(c) Solve $x\frac{dy}{dx} + y = x^3$ **[3mks]**

QUESTION 3

(a) Given the initial condition y(0)=1, find the particular solution of the equation $xydx + e^{-x^2}(y^2 - 1)dy = 0$ **[10mks]**

(b) Show that the following equation $(2x + y)dx + (x - 2y)dy = 0$ is homogeneous and hence or otherwise find its general solution. **[10mks]**

QUESTION 4

(a) Check for exactness and solve the equation.

$(5x^2 + 3x^2y^3 - 2xy^3)dx + (2x^3y - 3x^2y^2 - 5y^4)dy = 0$ **[15mks]**

(b) Newton's law of cooling states that if an object is hotter than the ambient temperature, then the rate of cooling of the object is proportional to the temperature difference

$$\frac{d\Theta}{dt} = -k(\Theta - A)$$

with $\Theta(t_0) = \Theta_0$, where $\Theta(t)$ is the object's temperature, A is the ambient temperature (a constant)and k is a positive constant. Show that by using the initial condition and rearranging, this is a first-order linear ODE results in, $\Theta(t) = A + (\Theta_0 - A)e^{-k(t-t_0)}$ **[5mks]**

QUESTION 5

(a) Find the general solution for $(x^2 - 2y^2)dx + xydy = 0$ **[10mks]**

(b) Given the logistic equation:

$$\frac{dp}{dt} = \mu p\left(1 - \frac{p}{p_\infty}\right)$$

with $p(t_0) = p_0$ solve the separable ODE and show that,

$p(t) = \frac{p_\infty}{1+(\frac{p_\infty}{p_0}-1)e^{-\mu(t-t_0)}}$ **[10mks]**