(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2016/2017 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR SCIENCE

COURSE CODE: **MAT 423**

COURSE TITLE: ODE II

DATE: 19/09/17 **TIME:** 11.30 AM -1.30 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

[Question One, 30mks]

(a) (i) Differentiate between an ordinary point and a singular point of the linear second order equation $y'' + p(x)y' + q(x)y = 0$ [3mks]

(ii) Write the following fourth order differential equation as a system of first order, linear differential equations

$y^{(4)} + 3y'' - \sin(t)y' + 8y = t^2; \quad y(0) = 1, y'(0) = 2,$

$y''(0) = 3, y'''(0) = 4$ [6mks]

(b) Write the following system in matrix form

$$\begin{aligned} x_1' &= 4x_1 + 7x_2 \\ x_2' &= -2x_1 - 5x_2 \end{aligned}$$

[4mks]

(c) how that $y = c_1 \cos 4x$ and $y = c_2 \sin 4x$, where c_1 and c_2 are arbitrary constants, are solutions of the differential equation

$y'' + 16y = 0.$ [4mks]

(d) Solve the following IVP

$$\begin{aligned} x_1' &= x_1 + 2x_2; \;\; x_1(0) = 0 \\ x_2' &= 3x_1 + 2x_2; \;\; x_2(0) = -4 \end{aligned}$$

[8mks]

(e) Prove that $x = 0$ is a regular singular point of the differential equation

$$x^2y'' + xy' + (x^2 - 4)y = 0$$

[5mks]

[Question Two, 20mks]

(a) Find the general solution to the following non- homogeneous system of differential equations using variation of parameter method.

$$\begin{aligned}\frac{dx}{dt} &= -3x + y + 3t\\ \frac{dy}{dt} &= 2x - 4y + e^{-t}\end{aligned}$$

on the interval $-\infty < t < \infty$ [8mks]

(b) Compute the eigenvalues and eigenvectors of the system

$$\begin{aligned}\frac{dx}{dt} &= 3x - 13y\\ \frac{dy}{dt} &= 5x + y\end{aligned}$$

[4mks]

(c) Determine the general solution of the following differential equation given that it is satisfied by the function $y = e^x$

$$xy'' - (x+1)y' + y = 0$$

[8mks]

[Question Three, 20mks]

(a) Compute the particular solution vector for

$$\begin{aligned}\frac{dx}{dt} &= 6x + y + 6t\\ \frac{dy}{dt} &= 4x + 3y - 10t + 4\end{aligned}$$

on the interval $-\infty < t < \infty$, hence find the general solution of the system. [9mks]

(b) Given that

$$\vec{X_1} = \begin{pmatrix} \cos t \\ -\frac{1}{2}\cos t + \frac{1}{2}\sin t \\ -\cos t - \sin t \end{pmatrix}$$

$$\vec{X_2} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} e^t$$

$$\vec{X_3} = \begin{pmatrix} \sin t \\ -\frac{1}{2}\sin t - \frac{1}{2}\cos t \\ -\sin t + \cos t \end{pmatrix}$$

Show that $\vec{X_1}, \vec{X_2}$ and $\vec{X_3}$ form a fundamental set of solution on the interval $(-\infty, \infty)$, hence find the general solution of the system on the interval. [5mks]

(c) Solve the differential equation $y' + y'' = x$ [6mks]

[Question Four, 20mks]

(a) Find the radius of convergence of the series

$$\sum_{n=0}^{\infty} \frac{(-3)^n (x-5)^n}{n 7^{n+1}}$$

[5mks]

(b) Verify that the Taylor Series for the function $f(x) = 3x^2 - 8x + 2$ about $x = 2$ is $3x^2 - 8x + 2 = -2 + 4(x-2) + 3(x-2)^2$ [5mks]

(c) Find a power series solution around $x_0 = 0$ for the following differential equation.

$$y'' + y = 0$$

[10mks]

[Question Five, 20mks]

Consider the ordinary differential equation

$$4xy'' + 2y' + y = 0$$

for which $x = 0$ is a regular singular point. Assuming a series solution of

$$y(x) = \sum_{n=0}^{\infty} a_n x^{r+n}$$

determine the values of r and a_n for $n \geq 0$.

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