(*Knowledge for Development*)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF EDUCATION AND

BACHELOR OF SCIENCE (MATHEMATICS)

COURSE CODE: **MAT 423**

COURSE TITLE: **ODE II**

DATE: 18/12/17 **TIME:** 8 AM -10 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 mks)

(a) Show that the solutions $\emptyset_1(x) = e^x \sin x$ and $\emptyset_2(x) = e^x \cos x$ to a differential equation are linearly independent. (4 mks)

(b) Linearize $y' = \frac{x}{y}$ at the point $y_0 = 1$. (4 mks)

(c) Use two methods to solve the equation $\frac{dx}{dt} = x + 1$; given $x(0) = 0$. (9 mks)

(d) Find the fundamental matrix for the system of equations given by $\boldsymbol{x}' = \begin{pmatrix} 5 & -1 \\ 3 & 1 \end{pmatrix} \boldsymbol{x}$. (10 mks)

(e) State : (i) the Existence Theorem (2 mks)

(ii) the Uniqueness Theorem. (1 mks)

QUESTION TWO(20 mks)

(a) The Bessel's equation of index k can be written as $y'' + \frac{1}{x}y' + \left(1 - \frac{k^2}{x^2}\right)y = 0$. When $k = \frac{1}{2}$ $y_1(x) = \frac{\sin x}{\sqrt{x}}$; $x > 0$ is a solution. If $y_2(x) = y_1(x) \int \frac{exp(-\int a(x)dx)}{y_1^2(x)} dx$ where $a(x) = \frac{1}{x}$, show that $y_2(x) = -\frac{\cos x}{\sqrt{x}}$ is another solution. (15 mks)

Hence prove that the solutions $y_1(x)$ and $y_2(x)$ are linearly independent.

(b) Verify that $y_1(x) = e^{2x}$ is a solution to the equation $y'' - 4y' + 4y = 0$. Hence use the formula given in (a) above to find another linearly independent solution. (5 mks)

QUESTION THREE (20 mks)

(a) By use of a suitable sketch diagram determine the stability of the function $f(x) = e^{-x} \sin x$ (7 mks)

(b) Linearize the following differential equations:

(i) $y' = y^3 + y;\ y_0 = 0$ (2 mks)

(ii) $y' = x^2y^2;\ y_0 = -2$ (2 mks)

(iii) $y' = -\sin y;\ y_0 = \frac{\pi}{4}$ (2 mks)

(c) The velocity $v = v(t)$ of a certain falling body subject to nonlinear velocity-dependent air resistance satisfies the equation $\frac{dv}{dt} = 32 - 0.01v^2$

(i)Find the linearization of this differential equation near $v_0 = 100$. (3 mks)

(ii) Solve the linearized equation found in part (i) subject to the initial condition $v(t) = 100$. (4 mks)

QUESTION FOUR (20 mks)

(a) If $\boldsymbol{x}(t) = c_1 \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^t + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2t}$ is a general solution , find a particular solution given that $\boldsymbol{x}(0) = \mathbf{I}$ (7 mks)

(b) Find the value of e^{At} when $\boldsymbol{x}' = A\boldsymbol{x}$ and $A = \begin{pmatrix} -3 & 2 \\ -4 & 3 \end{pmatrix}$ (13 mks)

QUESTION FIVE (20 mks)

(a)use Picard's method to approximate y when $x = 0.2$, given that $y = 1$ when $x = 0$ and

$\frac{dy}{dx} = x - y.$ (10 mks)

(b) Solve the system of linear differential equations given by:

$$2\frac{dx}{dt} + \frac{dy}{dt} - 4x - y = e^t$$

$$\frac{dx}{dt} + 3x + y = 0$$ (10 mks)