(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER

SPECIAL/SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE (MATHEMATICS)

COURSE CODE: MAT 421

COURSE TITLE: PDE I

DATE: 01/10/18 **TIME:** 8 AM -10 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

a) Using an example define what Partial Differential Equations (2 Marks)

b) Consider an equation of the form $F(x, y, z, a, b) = 0$ where a and b denote arbitrary constants and z is a function of x and y, explain how one can form a PDE from the equation (4 Marks)

c) Find the general solution of $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} + t\frac{\partial z}{\partial t} = xyt$ (6 Marks)

d) Find the Partial Differential Equation by eliminating the arbitrary constants m and n from $z = (x^2 + m)(y^2 + n)$ (5 Marks)

e) Find the complete integral of the equation $p(q^2 + 1) + (b - z)q = 0$ (5 Marks)

f) Solve

$$\frac{dx}{x(y^2 - z^2)} = \frac{dy}{-y(z^2 + x^2)} = \frac{dz}{z(x^2 + y^2)}$$

Using the multipliers $l_1 = x, m_1 = y, l_2 = \frac{1}{x}$ & $m_2 = -\frac{1}{y}$. (8 marks)

QUESTION TWO (20 MARKS)

a. Show that the equations $xp = yq, z(xp + yq) = 2xy$ are compatible and hence solve them. (14 marks)

b. Solve the following PDE $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} = \sin(x + 2y) + e^{2x+y}$ (6 marks)

QUESTION THREE (20 MARKS)

a. By eliminating the arbitrary functions, obtain the PDE from $z = f(x + ct) + g(x - ct)$ (7 Marks)

b. Find the integral surface of the PDE given by $z^2(p^2 + q^2) = x^2 + y^2$ (7 Marks)

c. Find the complete and general solution of $p + 4q = 7z + \tan(y - 4x)$ (6 Marks)

QUESTION FOUR (20 MARKS)

a) Find the integral surface of the linear PDE $x(y^2 + z^2)p - y(x^2 + 1)q = z(x^2 + y^2)$ which contains the straight line $x + y = 0,\ z = 1$. (6 Marks)

b) Solve $(D^2 + 2DD' + D'^2 - 2D - 2D')z = \sin(x + 2y)$ where $D = \frac{\partial}{\partial x}\ D' = \frac{\partial}{\partial y}$ (6 Marks)

c) Verify that the Pfaffian differential equation $yzdx + (x^2y - xz)dy + (x^2z - xy)dz = 0$ is integrable. (8Mks)

QUESTION FIVE (20 MARKS)

a) Prove the Lagrange's linear equation of the type

$$Pp + Qq = R$$

Where P, Q, R are functions of x, y, z and $p = \frac{\partial z}{\partial x}$ and $= \frac{\partial z}{\partial y}$. Use the arbitrary function $f(u, v) = 0$. Where u and v are functions of x, y, z (7 Marks)

b) Find the solution of the equation $(x^2 - 1)p + xyq + y^2z = x^2 - 1$ which is zero on the positive y –axis. In what region of the xy plane is the solution unique? (7 Marks)

c) Find the complete and general solution of $(x^2 - y^2 - z^2)p + 2xyq = 2xz$ (6 Marks)